

Decomposing and recomposing the population pyramid by remaining years of life

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The population pyramid evokes graphical inference about several aspects of demography that go beyond population structure itself. A top-heavy pyramid  suggests that the population is aged and aging, while a pyramidal pyramid  suggests that the population had and has high fertility. By association with high fertility we might infer high mortality. From other details in the profile we infer past wars and famines . Age-heaping  suggests an innumerate or superstitious population, and so we silently conclude that the population also has a lower average educational attainment, and expect to see a wide base. Such judgements are of course subject to the bias of the beholder, but we must admit that these are the kinds of thoughts that a pyramid evokes — the kinds of things we look for.

Age (and sex) is of technical use as the primary classifying variable for demographic rates. Rates are segmented by age because demographic phenomena show stable and consistent patterns over age; in order to make a statement about the population as a whole, it is held, one must purge rates of age structure. Of course, if age-specific rates themselves depend on age-structure, then we have a vicious circle, as Stolnitz and Ryder (1949) so eloquently pointed out. Age virtually always results significant in well-specified statistical models of demographic phenomena. We conclude that age is paramount to both parlour demography and serious demography.

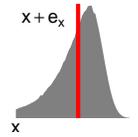
It therefore behooves the demographer to be explicit about the meaning of age. We use age to explain other phenomena, and these demographic forces lend phenomenological meaning back to age. Here we are just interested in the integer value itself. We all know that age is counted as the number of years that have passed since the event of birth. Age is in this way backward-looking. For this reason we consider the pyramid to be a reflection of the past. Age-structured demographic models therefore have this implicit connection to

the past, as do statistical inferences on age. That the lifeline traversing a Lexis diagram also has an endpoint signifying death is all too obvious, as is the idea that the length of this lifeline is proportional to the length of life. Time of birth is known, but time of death is not known  (until it's too late), and so we cannot measure lifelines directly for living populations.



The most widely applied approximation to lifeline endpoint is the so-called remaining life expectancy, e_x , which is an average remaining lifetime for the population of a given age, x , according to some particular assumptions about future mortality in the aggregate. Hersch (1944), Ryder (1975) and Sanderson and Scherbov (2007) have made good use of the remaining life expectancy column of the life table to gain insight into a population's potential future. The latter authors have elaborated a valid demographic perspective, that of remaining years of life, as a way of looking at age. Instead of counting up from birth, one may conceive of counting down until death — one's *prospective* age as opposed to chronological age. At least as early as 2001, Ken Wachter coined the term *thanatological* age¹ to refer to this manner of forward-looking age. These two terms are referring to the same thing: “Prospective” highlights the projective or potential nature of this view on age, while “thanatological” highlights the orientation along the lifeline.

Miller (2001) offers another tack on approximating remaining years of life for a given age group. Rather than assigning a single value, e_x , to all members in an age group, we recognize the full distribution over potential future death times, as inferred using information from the life table. This makes sense in general because it gives us more information, but also because the distribution around e_x of other possible death times is usually not symmetrical. When making generalizations about population aging based on remaining years of life, we obtain a closer approximation in summing over specific thanatological age-classes than by splitting population counts over e_x . Say we have single year population data, P_x and a radix-1 single year life table that describes the mortality experience of this population well-enough. Start either with survivorship, l_x , or the death distribution, d_x , and decompose P_x into the population of chronological age, x with remaining years of life, y , $P_{x,y}$:



$$P_{x,y} = P_x \cdot \frac{d_{x+y}}{l_x} \quad (1)$$

In doing (1) for each x and all possible values of y we have already decomposed P_x by remaining years of life. The result of this exercise is a matrix with population counts cross-classified by chronological age and thanatological age. One can blend out chronological age altogether:

$$P_y = \sum_{x=0}^{\omega} P_x \cdot \frac{d_{x+y}}{l_x} \quad , \quad (2)$$

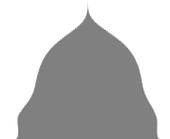
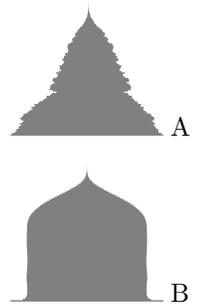
¹Thanatos was the Greek god of death.

which one might think of as recomposing what we have just decomposed by this new variable, y . If our assumptions about future mortality are not too far off, we have a fairly good approximation of thanatological age structure, P_y .

A great many practical and theoretical results will likely fall out of playful interaction with thanatologically transformed data, some of which have been spelled-out in Riffe (2013). Here we interest ourselves directly with the visual inspection of this decomposition-recomposition — in how remaining lifetimes are distributed over age and how chronological age is distributed over remaining lifetimes — via the population pyramid.

Our objective is to build the same kind of demographic associations with thanatological age-structure that we get from looking at traditional population pyramids. At a glance, what does a given profile suggest about the past and future of mortality and fertility for a given population? The two population structure profiles in the margin, A and B, compare our two orientations of age for a particular year and population. One recognizes thanatological population structure first for its relative smoothness. An artifact of the transformation method is that roughness is blended out due to the staggering of ages. Of the two pyramids in the margin, B is much smoother than A. The traditional pyramid, A, represents a population that either is growing very rapidly, or that has high fertility and high attrition, or in this case some of both. The extruding bottom rung of B indeed contains all ages, but this kind step only arises when the first year or two of life has high mortality. From B, we see that if vital conditions persist, the relative size of attrition (at least for those beyond infancy) does not really change from year to year, despite the aging of large cohorts at the bottom of the age pyramid, A. If conditions are roughly constant, we conclude that the growth rate for this population has been a bit higher than zero. Of course, such intuition can lead us astray, as a population's vital rates may be in flux. A and B both represent 1900 Sweden, with male and female life expectancies at birth of 50.79 and 53.62, mean remaining life expectancies of 46.36 and 43.12 (the weighted average of thanatological age), and mean (chronological) ages of 34.74 and 36.32, respectively. This was a young and growing population, at least according to its period indicators.

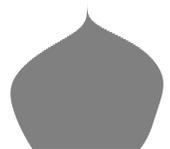
A mortality crisis year, such as 1918, will cause an abrupt sag in the thanatological age structure with respect to surrounding years. WWI and WWII caused the male (left) side to sag more than the female side in some countries, for a rather lop-sided pyramid. In recent years, the typical profile of contemporary thanatological age-structure for most low-mortality countries resembles the average. Populations that have recently undergone a rapid mortality decline, but are still young will have very narrow bases and very high centers of gravity. All contemporary (postwar) populations have maintained such a taper at the base — instead mortality gains have tended to both slow and disperse the advancement of large birth cohorts toward the lowest thanatological ages. These profiles can be enriched to yield more information.



SE, 1918

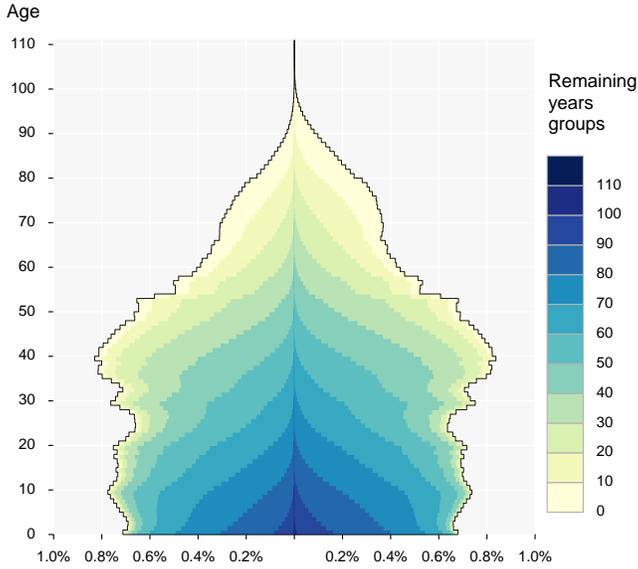


ENW, 1944

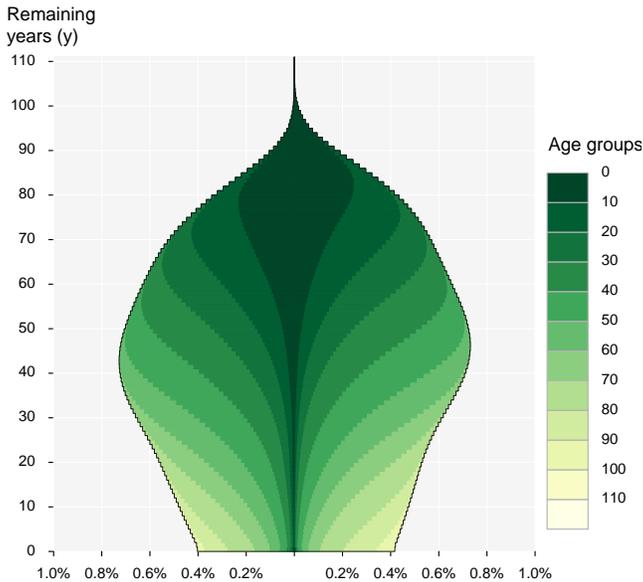


Avg. contemp.

US chronological age structure, 2000, with thanatological age-groups indicated by shades (HMD)



(below) US thanatological age structure, 2000, with chronological age-groups indicated by shades (HMD)



²For the botanically-minded, the thanatological age-sex-structure of low-mortality populations almost always takes the shape of a truncate deciduous leaf with entire margins, and, when chronological age is also highlighted in the leaf, as here, arcuate venation.

The outer profile of the first pyramid is perhaps familiar to the reader as the year 2000 US population. By following (1) and then aggregating y into 10-year groups within each x , we can add a layer of information to this figure. One gains an overview of the heterogeneity of remaining lifetimes within each age. This exercise offers a manifold gain in resolution for the calculation of indicators on population aging, health, indirect indicators of dependency, and even just a simple idea of the likely distribution of future lifetimes for a given age-group. According to the color gradient used here, a darker pyramid indicates a population with many years to live and vice versa. If temporal proximity to death can be thought of as a useful indicator of morbidity or disability, one might here gauge how these states are distributed over age.

The second pyramid is based on the transformation equation (1) as well, but instead we have turned the population on its side and aggregated chronological age into 10-year groups. Now we see single-age resolution of thanatological age-structure (purely probabilistic), and an idea of the time-since-birth heterogeneity within each remaining lifetime. The bottom layer in this depiction, which resembles a leaf², is expected to decrement within the year. A top-heavy leaf indicates a population whose members have many years ahead. According to our gradient, a dark leaf is young and a light leaf is older. Population leaves are sensitive to mortality conditions, and history has known them to take different characteristic shapes.

*This is a work in progress. Results to be expanded upon with more comparisons of populations under different age structures and mortality conditions, including developing countries.

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