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Measurement and Estimation of Nutritional Deficit Among Children in Developing Countries: An Extension of Mora's Method using Skew Normal Distribution

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Abstract

The World Health Organization (WHO) classification scheme for the degree of prevalence of malnutrition in a population suggests using anthropometric Z-scores with a cut off value of -2 to see the percentage of subjects belonging to the study population lying two standard deviations below the reference median and quantify the prevalence using this percentage. Although anthropometric indicators are widely used for assessing the nutritional status of children, lack of consensus on standardized growth reference and the choice of cut-off points for prevalence estimates have restricted the use of such measures particularly in the developing countries. Towards this goal, the present article proposes an extension of Mora's method (Mora, 1989) to estimate nutritional deficit using skew normal distribution for the distributions of the Z-scores of the observed populations. The application of the proposed method is illustrated for the selected states/regions from three countries, India, Bangladesh and Nepal using latest NFHS/DHS survey data.

Keywords: Anthropometric z-scores, Box–Cox Transformation, location and scale family, reference population, skew normal distribution.

1. INTRODUCTION

Child malnutrition is internationally recognized as an important public health indicator and an essential component of a country's overall human development. The devastating effects of malnutrition on human performance, health, and survival are today well established (Chang et al, 2002; Martorell, 1992; Walker et al, 2000; Mendez and Adair, 1999; Pelletier and Frongillo, 2003; Caulfield et al, 2004) and a recent global analysis demonstrated that child malnutrition is the leading cause of the global burden of disease (WHO, 2002). As a result of the increased recognition of the relevance of nutrition as a basic pillar for social and economic development, monitoring trends in childhood malnutrition has gained increasing importance in assessing the progress made by nations in achieving internationally set goals, such as the Millennium Development Goals.

It is widely accepted that for practical purposes anthropometry is the most useful tool for assessing the nutritional status of children. Admittedly almost any illness will impair a child's growth, but in practice in developing countries growth deficits are caused by two preventable factors, inadequate food and infections. In general, infections influence body size and growth through their effects on metabolism and nutrition. Thus, the classical use of anthropometry as the most readily available method of assessing nutrition is logical and is popularly practiced (WHO, 1986). It is also recognized that, a deficit in growth is not necessarily only due to inadequate nutrition and chronic ailments, genetic factors, both within and between populations may also affect growth. In spite of all these caveats, the central role of anthropometry in nutritional assessment is widely accepted and used as a tool to estimate the nutritional status of populations and to monitor the growth and health of children and adults.

Anthropometric assessment for children and adolescents involves the use of growth standards and/or growth references for assessing their growth, nutritional status and well being (Wang and Chen, 2012 ; WHO, 1995). A growth standard reflects optimal growth, suggesting that all children have the potential to achieve that level, while a growth reference is simply the distribution used for comparison (WHO,2006). Use of a single international reference population allows comparison of results among different nutrition and health studies and, thus, greatly assists the interpretation of results. The World Health Organization (WHO) proposed in 1978 that a single anthropometric growth reference be used both for individual child growth monitoring and for assessing the nutritional status of populations (WHO,1978). Irrespective of the reference population used, an anthropometric indicator provides a measure of an individual's growth status in relation to the reference median, expressed either as a percentile, a percentage of the reference median, or as a proportion of the standard deviation often referred to as a Z-score. The use of a reference population makes it possible to compare the growth status of children of different ages and makes it feasible to assess anthropometric status in population studies and in surveillance programs. Percentiles and *Z* -scores in anthropometric measures have been widely used to help assess young people's nutritional status and growth, such as undernutrition (e.g., underweight, stunting and wasting) and overnutrition (i.e., overweight and obesity). Often, percentiles (such as the 5th, 85th, 95th, 97th, 99th percentiles) and *Z* -scores (e.g., -2 and +2) are used to classify various health conditions, and sex-age-specific anthropometric measures cut-points (based on *Z* -scores or percentiles).

A large number of studies in the last three decades measured child nutritional status in developing countries since the introduction of 1978 WHO/NCHS (National Centre for Health Statistics) reference growth charts (WHO, 1978) which was based on data on weight and height from a statistically valid sample of infants and children in the United States. The use of these charts, however, has never been unanimously accepted (especially for children aged 0–24 months) neither for use within the U.S., nor as an international standard. Detractors have called attention to several limitations, perhaps more importantly the sample used for their construction and in particular, concerning the extent to which growth paths depend on feeding practices. As a result of the several shortcomings with the 1978 WHO/NCHS reference growth charts, WHO initiated the Multicentre Growth Reference Study (MGRS), which between 1997 and 2003 produced a new set of references for international comparisons that has become recently available (WHO MGRS, 2006). The new charts are based on healthy children from Brazil, Ghana, India, Norway, Oman and United States, living under conditions likely to allow the full achievement of their genetic growth potential, whose mothers did not smoke and followed the WHO recommended feeding practices. The latest round of the National Family and Health Surveys (NFHS-3, 2005-06) in India has used this new international reference population released by WHO in April 2006 (WHO Multicenter Growth Reference Study Group, 2006) and accepted by the Government of India.

The use of Z -scores is recommended for several reasons. First, Z -scores are calculated based on the distribution of the reference population (both the mean and the standard deviation); thus, they reflect the reference distribution. Second, as standardized measures, Z -scores are comparable across age, sex and measure (as a measure of "dimensionless quantity"). Third, a group of Z -scores can be subject to summary statistics such as mean and SD and can be studied as a continuous variable. In addition, Z -score values can quantify the growth status of children outside of the percentile ranges (WHO, 1995). As has been mentioned earlier, Z -scores of +2 and -2, have often been chosen as cut points to classify problematic growth/nutritional status such as undernutrition or obesity. These criteria are somewhat arbitrary as they are based on statistical distribution rather than on the risks of health outcomes.

Despite the popularity and recognized usefulness of nutritional anthropometry in assessments of health and nutrition, there have been many discussions and conflicting recommendations about the cut-off points to be used for estimating the prevalence of undernutrition. Different cut-off points and classification systems have been proposed and used for estimating the prevalence of malnutrition in population surveys; thus the reported rates are often not comparable and sometimes questionable. This confusion and the consequent lack of standard analytical methods have apparently legitimized an unfortunate tendency to leave every country open to set up its own criteria, depending on the local circumstances, for the sake of practicability.

The present article proposes a method for estimating the prevalence of undernutrition in a population using the anthropometric z-score measurements available from cross-sectional population surveys, which is particularly suitable for the developing countries. The methodology presented here is an extension of the work by J.O.Mora (Mora, 1989) in which the conflicting choices of cut-offs were resolved. The paper is organized as follows. Section 2 describes the various anthropometric z-scores, how are these computed and WHO classification scheme for prevalence of malnutrition. Section 3 discusses the method proposed by J.O.Mora and its limitations regarding its usuability for the developing countries. We have carried out exploratory analysis of sample z-scores in section 4 for selected Indian states where an attempt is made to show that the anthropometric data are substantially nonormal. Section 5 builds up a probability model for the z-scores based on the empirical findings and finally, in section 6, a model based measure of nutritional deficit is obtained and empirical illustrations are provided.

2. Anthropometric z-scores and WHO classification scheme

Three commonly used anthropometric indices are derived by comparing height and weight measurements with reference curves: height-for-age, weight-for-age, and weight-for-height. Although these indices are related, each has a specific meaning in terms of the process or outcome of growth impairment. Deficits in one or more of the anthropometric indices are often regarded as evidence of "malnutrition". The ranges of the deficit of physical status based on each index vary significantly across populations. The most common approach to evaluate growth performance transforms the anthropometric indices into a Z-score (standard deviation scores). The Z-score system expresses the anthropometric value as a number of standard deviations or Z-scores below or above the reference mean or median value. A fixed Z-score interval implies a fixed height or weight difference for children of a given age. For population-based uses, amojor advantage is that a group of Z-scores can be subjected to summary statistics such as the mean and standard deviation. As we shall see shortly, that the distributions of Z-scores are considered for the purposes of such comparisons.

Let g denote the reference group a child is being compared to, and let x_{ig} represent weight or height of a specific child i in a group g. When the indicator measures height, the group is defined by age and gender. When the indicator measures weight, the reference group is identified by gender and either age (in the case of weight-for-age) or height (in the case of weight-for-height). To gauge the nutritional status of a child, it is necessary to compare the child's outcome to a corresponding "normal/standard / reference" outcome for a child that belongs to the same group.

Defn. 1. (WHO/NCHS 1978)

$$Z_{ig} = \frac{x_{ig} - \bar{x}_g}{s_g}$$

where $\bar{x}_g$ and s_g are, respectively, the mean (or median) and the standard deviation of the indicator for children within the same group in the benchmark population. For several years, the WHO/NCHS 1978 charts have represented the most widely used reference to assess child nutritional status. Their use became especially common after the WHO recommended their use for the evaluation of child growth worldwide (Waterlow et al., 1977; World Health Organization, 1978; Dibley et al., 1987a). When the corresponding nutritional indicator in the reference population is approximately normally distributed, z-scores are very easy to interpret. For instance, if a boy's weight-for-height z-score lies below -1.645 then his weight is below that of 95% of boys in the reference population with the same height. A child is usually identified as stunted if height-for-age z-score is below -2, and as underweight or wasted if the z-score for, respectively, weight-for-age or weight-for-height is below the same threshold. In India, the WHO/NCHS 1978 charts have also been adopted for the calculation of the z-scores (NFHS 2, 1996-97).

The appropriateness and use of this WHO/NCHS 1978 reference standard was being debated for children in developing countries, in particular concerning the extent to which growth paths depend on feeding practices. Growing concerns over the use of this reference has led to a revision of the charts used for international comparisons (WHO 2006). Unlike the old references, the new charts have been developed using an LMS model (Cole, 1988; Cole and Green, 1992), which takes explicitly into account the skewness and non-normality of the distribution of weight and height in the reference population. In this approach, the z-score for a given anthropometric measure x_{ig} is calculated using mean and standard deviation not of the same measures in the reference group, but of a Box–Cox transformation of the measures. In this way, the z-score for child i compared to a reference group g is calculated as following.

Defn.2. (WHO 2006)

$$Z_{ig} = \frac{\left(x_{ig}/M_g\right)^{L_g} - 1}{L_g S_g}$$

where Lg is the ‘‘power’’ of the Box–Cox transformation and Mg and Sg are the mean and the standard deviation of the transformed variable in the reference population. Hence, the new charts provide the parameters Lg, Mg and Sg necessary for the calculation of the above expression. Such parameters are gender-age specific for the construction of height-for-age and weight-for-age z-scores, while they are gender-height specific for weight-for-height.

Using z-scores, the most commonly used cut-off to define abnormal anthropometry is a value of –2, that is, two standard deviations below the reference median, irrespective of the indicator used. For example, a child whose height-for-age z-score is less than –2 is considered stunted. This provides the basis for estimating prevalence of malnutrition (POM) in populations or subpopulations. World Health Organization has also proposed a classification scheme for POM. Accordingly, POM in populations or subpopulations is assessed by referring to a classification scheme recommended by WHO (WHO,1995) which is reproduced below in Table-1.

Table 1. WHO Classification Scheme for Degree of Population Malnutrition

	Prevalence of malnutrition (% of children <60 months, below –2 z-scores)	
Degree of malnutrition	**Weight for Age / Height for Age**	**Weight for Height**
Low	<10	<5
Medium	10–19	5–9
High	20–29	10–14
Very high	≥30	≥15

Source: WHO 1995.

3. Methodological Issues

Malnutrition refers to all deviations from adequate nutrition, including undernutrition (and overnutrition or obesity) resulting from inadequacy of food (or excess of food) relative to need (respectively). Malnutrition also encompasses specific deficiencies (or excesses) of essential nutrients such as vitamins and minerals. Conditions such as obesity, although not the result of inadequacy of food, also constitute malnutrition. The terms "malnutrition" and "undernutrition" are often used loosely and interchangeably, although a distinction is, and needs to be, made at all times. In the present article, we shall mostly restrict our attention to the presence of abnormal anthropometry that is indicative of undernourishment which is the thrust of the MDGs in the developing world.

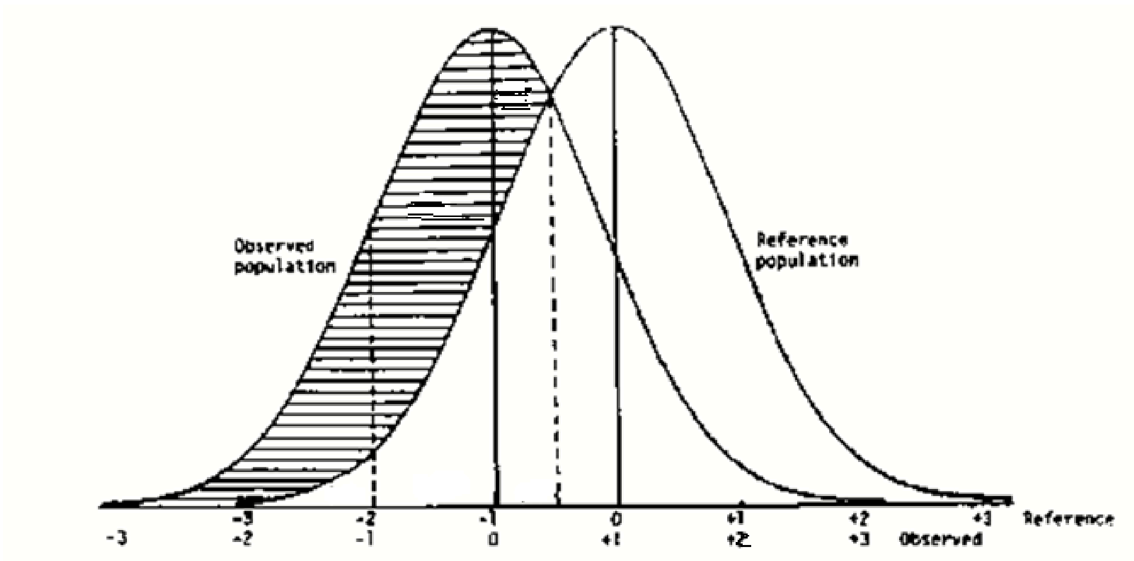
Suppose a population of size N is composed of those D who are diseased and N – D who are not. In the folklore of epidemiology, the prevalence is defined as $p = \frac{D}{N}$. Similarly, the prevalence of childhood malnutrition, is defined as the proportion or percentage of children under age five having abnormal anthropometry beyond certain thresholds or cut-offs. WHO proposed that the normal range for any population should be between plus and minus two standard deviation (± 2 S.D.) units of the median, a range that includes 95.4 per cent of the reference population, and would yield only about 2.3 per cent false positives on each side.

Besides, its statistical justification, the WHO cut-off point, below which the values are seen as potentially abnormal, has been further supported by studies of "functional outcomes" showing a significant increase in the risk of mortality (Klelmenn and McCord, 1978; Chen et al, 1980), as well as a decreased immune response when anthropometric indicators drop below such a threshold (Reddy et al, 1976). Thus, the WHO recommendation has been generally adopted and the cut-off point at two standard deviation units below the reference median has been widely accepted and used lately for estimating the prevalence of malnutrition in national surveys. Further, anticipating that a cut-off point of two standard deviations might dramatically lower prevalence rates of malnutrition in developing countries and might tend to underestimate the magnitude of the problem, WHO then suggested (WHO, 1983), as part of its methodology for measuring change in nutritional status, using either one or two standard deviations as the dividing line between normality and abnormality, but adjusting the resulting prevalence by subtracting the proportion of cases expected below such a cut-off point in the normal distribution.

Clearly, conflicting recommendations about the cut-off points to be used for estimating the prevalence of anthropometric abnormality have profound implications and might be misused for non-scientific purposes (Mora, 1989). A floating cut-off point could indeed be moved up or down depending on whether the interest is to dramatize the seriousness of the problem or to show that it is of much less magnitude. Keller (Keller, 1983) contended that "if reasonable simple statistical methods were available, it would be more desirable to compare distributions rather than prevalences, which to some degree distort biological realities". Following this, J. O. Mora (Mora, 1989), had developed a simple method for estimating a standardized prevalence of child malnutrition from anthropometric indicators by comparing the observed and the reference distribution as suggested by Keller. The method was based on the assumption that both the distributions are normal and under optimal environmental conditions everybody will grow within the boundaries of the reference population distributions, so that all individuals growing outside of this distribution do so because of environmental constraints. Suppose, the standardized Z-score of the observed population or sub-population follows a normal distribution with mean μ and standard deviation σ. The estimated standardized prevalence (SP) as proposed by Mora can be obtained by using the following formula

$$SP = \begin{cases} \Phi\left(\frac{\mu - \sigma\sqrt{\mu^2 + 2\sigma^2 \ln\sigma - 2\ln\sigma}}{1-\sigma^2}\right) + \Phi\left(\frac{\mu\sigma - \sqrt{\mu^2 + 2\sigma^2 \ln\sigma - 2\ln\sigma}}{1-\sigma^2}\right), for\ \sigma > 1 \\ 2\Phi\left(\frac{\mu}{2}\right) - 1, for\ \sigma = 1. \end{cases}$$

It was shown that for a given distance between curves, the same standardized prevalence rate is found irrespective of the cut-off points and as such, the prevalence is a function of the distance between the observed and the reference curve and not of the cut-off points. When the standard deviation of the observed distribution is also one, the estimated standardized prevalence is represented by the shaded area in Fig. 1, i.e. by that portion of the observed distribution that is uncovered by the reference population distribution. It is for this reason, we shall prefer to call it by nutritional deficit rather than standardized prevalence. The author also provided a table to facilitate a rapid assessment of standardized prevalence whenever the observed distribution of Z-scores is approximately normal.



Standard Deviations in standardized z-scores of the reference population distribution

Fig. 1. Overlapping of the Gaussian distributions of an anthropometric indicator in the observed and in the reference population. The shaded area represents the standardized prevalence of abnormality in the observed population.

A major limitation of the procedure discussed above is that the assumption of normality is untenable for the distributions of Z-scores when observed from the developing countries. As we shall proceed through the next section we shall examine this issue using a battery of statistical test procedures.

4. Exploratory Analysis of Anthropometric z-scores

For the purpose of exploratory analysis, we have considered data on the three anthropometric indices, height for age, weight for age, and weight for height from three countries India (NFHS 3, 2005-06), Bangladesh (DHS, 2011), and Nepal (DHS, 2011). Two states were selected from each of the selected countries initially for this purpose. We have calculated (Table 2) univariate descriptive and robust statistics in order to explore features of the empirical distribution of the z-scores. First three columns of each of these tables provide mean, 5% trimmed mean and median of the empirical distribution of each of the z-scores. Near equality of these values for a given score would mean a symmetric distribution and otherwise with a skewness. In most of states, all the three scores display a higher value of mean than either the trimmed mean or median. It may be that the outliers or extreme values are responsible to pull it upwards. The skewness and kurtosis statistics also provide evidence of disproportionate values at the tails of the respective distributions. In few cases, such as, for Meghalaya and Uttar Pradesh, the Weight for Age and Weight for Height z-scores give evidence of a negative skewness. The so called descriptive statistics are followed by a skewness-kurtosis plot that helped to select which distribution(s) to fit among the potential candidates. Figure 2 provides a skewness-kurtosis plot as the one proposed by Cullen and Frey (1999). On this plot, values for common distributions are also displayed to help the choice of distributions to fit to data. In order to take into account the uncertainty of the estimated values of kurtosis and skewness, the data set was bootstrapped 5000 times. The values of skewness and kurtosis corresponding to the bootstrap samples are then computed and reported in blue color on the skewness-kurtosis plot.

There exists a vast literature on tests of normality and their statistical properties (Lilliefors, 1967; Shapiro and Wilk, 1965; Jarque and Bera, 1980; D'Agostino and Stephens, 1986). The most popular omnibus test for normality for general use is the Shapiro-Wilk (SW). The Jarque-Bera (JB) test is the most widely adopted omnibus test for normality in economics and related field. The Lilliefors (Kolmogorov-Smirnov) (L(KS)) test is the best known omnibus test based on empirical distribution function. Being omnibus procedures, SW, JB, L(KS) and many others do not provide insight about the nature of deviations from normality e.g. skewness, heavy tails or outliers. Therefore, specialized tests directed at particular alternatives are desired in many practical situations. In this article, tests of normality (Table 3A) were carried out using various omnibus procedures when possible. These omnibus tests are accompanied by various tests (Table 3B and 3C) of symmetry and directed tests of normality against heavy tailed alternatives and outliers (Gel et. al. 2006). Each of these tests are also followed by four goodness of fit plots (figure 2, lower panel). All of the results are not reproduced here to save the space. The tests of normality overlay a normal curve on actual data, to assess the fit. A significant test means the fit is poor. In majority of the cases, our analyses show that normality is an untenable assumption or it is reasonably poor. Results as shown in table 3C suggests that although the symmetry in many of the cases are evident, the empirical distributions are characterized by heavier tails or outliers and are substantially nonnormal.

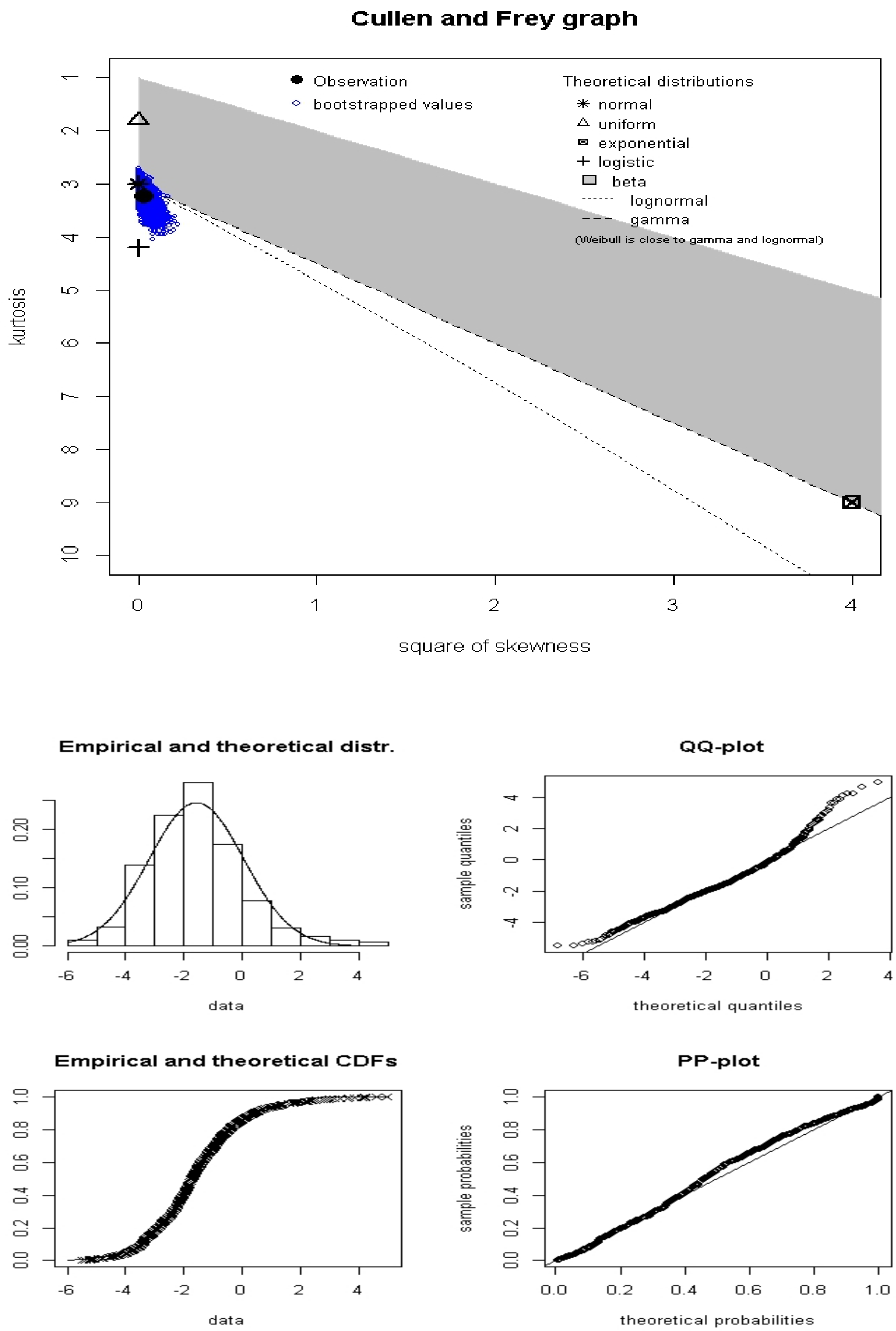


Figure 3. Cullen and Frey plot (upper panel). Goodness of fit plots (lower panel).

Table 2. Descriptive and robust statistics for z-scores

Height for Age									
Country	States/ Regions	mean	5% trimmed mean	median	sd	skewness	kurtosis	% below -2sd	Number of children (weighted)
India[1]		-1.87	-1.90	-1.93	1.66	0.398	0.784	48.0	45377
	Kerala	-1.1	-1.14	-1.28	1.42	0.52	1.36	24.5	896
	Uttar Pradesh	-2.18	-2.23	-2.26	1.68	0.46	0.68	56.8	5597
Bangladesh[2]		-1.67	-1.70	-1.72	1.40	0.384	1.357	41.3	9217
	Barisal	-1.73	-1.77	-1.79	1.46	0.668	2.364	43.2	1009
	Dhaka	-1.67	-1.70	-1.77	1.42	0.471	1.567	42.6	1533
Nepal[3]		-1.71	-1.72	-1.75	1.37	0.167	0.396	40.5	2335
	Eastern	-1.62	-1.63	1.66	1.29	0.074	0.446	39.1	534
	Central	-1.58	-1.59	-1.62	1.40	0.170	0.259	38.2	482

Weight for Age									
Country	States/ Regions	mean	5% trimmed mean	median	sd	skewness	kurtosis	% below -2sd	Number of children (weighted)
India		-1.78	-1.79	-1.78	1.23	0.144	0.422	42.5	45377
	Kerala	-1.22	-1.24	-1.23	1.08	0.21	0.35	22.9	896
	Uttar Pradesh	-1.83	-1.83	-1.80	1.19	-0.01	0.27	42.4	5597
Bangladesh		-1.61	-1.63	-1.64	1.15	0.275	0.816	36.4	9217
	Barisal	-1.63	-1.65	-1.71	1.14	0.432	0.654	38.5	1009
	Dhaka	-1.56	-1.59	-1.58	1.20	0.381	0.984	35.6	1533
Nepal		-1.46	-1.46	-1.48	1.10	0.064	0.299	28.8	2335
	Eastern	-1.33	-1.33	-1.33	1.03	0.034	0.241	25.5	534
	Central	-1.38	-1.38	-1.41	1.11	0.138	0.173	28.6	482

Weight for Height									
Country		mean	5% trimmed mean	median	sd	skewness	kurtosis	% below -2sd	Number of children (weighted)
India		-1.02	-1.02	-0.99	1.29	0.094	1.015	19.8	45377
	Kerala	-0.88	-0.88	-0.86	1.23	0.096	0.16	15.9	896
	Uttar Pradesh	-0.58	-0.55	-0.47	1.31	-0.34	0.88	14.8	5597
Bangladesh		-0.93	-0.94	-0.96	1.21	0.231	1.296	15.6	9217
	Barisal	-0.90	-0.93	-0.97	1.15	0.406	1.147	14.3	1009
	Dhaka	-0.86	-0.88	-0.90	1.26	0.266	1.014	16.3	1533
Nepal		-0.67	-0.67	-0.66	1.12	0.002	0.745	10.9	2335
	Eastern	-0.58	-0.59	-0.60	1.12	0.205	0.698	10.3	534
	Central	-0.66	-0.64	-0.69	1.14	-0.097	0.379	11.4	482

Source: Author's own calculation. [1]NFHS 3(2005-06), [2]DHS 2011, [3]DHS 2011

Table 3A. Omnibus Tests of Normality for z-scores

Height for Age						
State	Shapiro-Wilk (p-value)	Anderson-Darling (p-value)	Cramer-von Mises (p-value)	Lilliefors(K-S) (p-value)	Pearson chi-square (p-value)	Classical Jarque-Bera (p-value)
Kerala	0.977 (0.000)	5.639 (0.000)	0.953 (0.000)	0.058 (.000)	73.30 (0.000)	108.84 (0.000)
Uttar Pradesh	NA	9.234 (0.000)	1.410 (0.000)	0.031 (.000)	217.43 (0.000)	283.0 (0.000)
Barisal	0.976 (0.000)	1.965 (0.000)	0.291 (0.000)	0.044 (0.000)	29.99 (0.315)	214.83 (0.000)
Dhaka	0.981 (0.000)	3.660 (0.000)	0.596 (0.000)	0.038 (0.000)	61.36 (0.001)	194.57 (0.000)
Eastern	0.996 (0.243)	0.383 (0.396)	0.047 (0.558)	0.023 (0.720)	25.08 (0.293)	4.609 (0.099)
Central	0.995 (0.136)	0.671 (0.079)	0.109 (0.084)	0.038 (0.092)	26.28 (0.196)	3.512 (0.173)

Table 3B. Tests of Symmetry for z-scores

Height for Age				
State	MGG (p-value)	Mira (p-value)	Cabilio-Masaro (p-value)	D'Agostino (p-value)
Kerala	5.119 (0.000)	4.814 (0.000)	4.841 (0.000)	3.995 (.000)
Uttar Pradesh	5.04 (0.000)	5.05 (0.000)	4.92 (0.000)	8.583 (.000)
Barisal	1.02 (0.308)	0.980 (0.327)	0.969 (0.333)	4.398 (0.000)
Dhaka	3.597 (0.000)	3.459 (0.000)	3.435 (0.000)	4.666 (0.000)
Eastern	1.028 (0.311)	1.005 (0.315)	1.012 (0.311)	0.466 (0.641)
Central	0.841 (0.401)	0.826 (0.409)	0.819 (0.413)	1.011 (0.312)

Table 3C. Robust Directed Tests of Normality against Heavy-tailed alternatives

Height for Age				
State	Bonett-Seier (p-value)	SJ Test (p-value)	Robust Jarque-Bera (p-value)	Anscombe-Glynn (p-value)
Kerala	5.418 (0.000)	6.447 (0.000)	138.98 (0.000)	5.314 (0.000)
Uttar Pradesh	5.967 (0.000)	6.398 (0.000)	299.2 (0.000)	7.549 (0.000)
Barisal	5.309 (0.000)	5.541 (0.000)	209.87 (0.000)	6.789 (0.000)
Dhaka	5.791 (0.000)	6.333 (0.000)	202.09 (0.000)	6.989 (0.000)
Eastern	1.249 (0.212)	1.377 (0.096)	3.945 (0.139)	1.877 (0.061)
Central	2.035 (0.07)	2.205 (0.023)	5.317 (0.07)	1.163 (0.245)

Source: Author's own calculation

5. Modeling z-scores with skew normal distribution

The skew normal distribution, proposed by Azzalani (1985), can be a suitable model for the analysis of data exhibiting a unimodal density having some skewness present, a structure often occurring in data analysis. The proposed distribution is a generalization of the standard normal distribution and the probability density function (pdf) is given by

$$f(z;\lambda) = 2\phi(z)\Phi(\lambda z), \quad -\infty < z < \infty, \qquad (5.1)$$

where $\phi(x)$ and $\Phi(x)$ denote the N(0,1) density and distribution function respectively and we write Z~SN(λ). The parameter λ regulates the skewness and $\lambda = 0$ corresponds to the standard normal case. The density given by (5.1) enjoys a number of formal properties which resemble those of the normal distribution, for example, if X has the pdf, given by (5.1), then X^2 has a chi-square distribution with one degree of freedom. That is, all even moments of X are exactly the same as the corresponding even moments of the standard normal distribution. For more information, see Azzalini and Capitanio (1999) and Genton (2004). A motivation of the skew normal distribution has been elegantly exhibited by Arnold et al. (1993). This model can naturally arise in applications as hidden function and/or selective reporting model, see Arnold and Beaver (2002).

A more flexible and suitable class of distributions (Pourahmadi, 2007) incorporating the skewness factor, is a location and scale extension of the family defined in (5.1). Let us denote the location and scale parameters by ξ and ω respectively, then for any Z~SN(λ), define a general SN random variable by

$$Y = \xi + \omega Z, \qquad (5.2)$$

and write, Y~SN(ξ, ω, λ) for this random variable. The pdf of Y can shown as

$$f(y;\ \xi,\omega,\lambda) = \frac{2}{\omega}\,\phi\left(\frac{y-\xi}{\omega}\right)\Phi\left(\lambda\frac{y-\xi}{\omega}\right) \qquad (5.3)$$

and the moment generating function of Y is given by

$$M_Y(t) = E(e^{tY}) = 2exp\left(\xi t + \frac{\omega^2 t^2}{2}\right)\Phi(\delta\omega t), \qquad (5.4)$$

where $\delta = \lambda/\sqrt{1+\lambda^2}\ \epsilon(-1,1)$. From (5.4), it follows that:

$$E(Y) = \xi + \omega\mu_z,\ var(Y) = \omega^2(1-\mu_z^2),\ \ \gamma_1 = \frac{4-\pi}{2}\,\frac{{\mu_z}^3}{(1-{\mu_z}^2)^{3/2}},$$

and $\gamma_2 = 2(\pi-3)\frac{{\mu_z}^4}{(1-{\mu_z}^2)^2}$, where $\mu_z = \delta\sqrt{2/\pi}$, γ_1 and γ_2 denote the standardized third and fourth-order cumulants respectively.

We have carried out the fitting of SN(ξ, ω, λ) using the software library 'sn' version 0.4-17 (Azzalini, 2011). The function 'sn.em' is used for this purpose which is based on EM algorithm to locate the maximum likelihood estimates. The estimates obtained using this function are very robust, although it generally takes a longer computation time. Table 4 gives the maximum likelihood estimates of the parameters, the estimates of mean, sd and, skewness. We also provide two values of log-likelihood at convergence; the first one corresponds to the default setting in which a global maximization is performed, and the second corresponds to the setting in which the shape parameter is fixed at 0. The maximum of the two leads to the choice of a SN distribution over a normal model.

6. Measurement and Estimation of nutritional deficit

We assume that the probability distribution of an anthropometric z-score follows a location-scale skew normal distribution i.e. Y~SN(ξ, ω, λ) with pdf given by (5.2). The advantage of such an assumption is not only their close fit to the sampled data on z-scores, but also that the reference population, with which it is usually compared to measure the degree of prevalence of malnutrition, belongs to the same family and thus facilitates the comparison. More importantly, the methodology proposed by Mora (Mora, 1989), is thus generalized here, for possible applications when the distributions of the Z-scores for the observed populations/sub-populations are not normal.

Table 4. Skew normal goodness of fit statistics for z-scores

Height for Age								
State	Parameter estimates			Estimates of			Log-likelihood	
	location	scale	shape	mean	sd	skewness	SN	normal
Kerala	-2.43	1.94	1.70	-1.09	1.41	0.36	-1569.7	-1585.4
Uttar Pradesh	-3.72	2.36	1.72	-2.09	1.71	0.37	-10223.4	-10297.8
Barisal	-3.03	1.97	1.55	-1.71	1.46	0.32	-1495.4	-1508.2
Dhaka	-2.94	1.93	1.52	-1.66	1.44	0.31	-2255.6	-2272.5
Eastern	-2.37	1.49	0.82	-1.62	1.29	0.09	-891.4	-891.8
Central	-2.59	1.72	1.09	-1.58	1.39	0.16	-842.8	-843.9

Weight for Age								
State	Parameter estimates			Estimates of			Log-likelihood	
	location	scale	shape	mean	sd	skewness	SN	normal
Kerala	-2.04	1.35	1.18	-1.22	1.08	0.19	-1333.375	-1336.60
Uttar Pradesh	-2.30	1.34	0.62	-1.73	1.22	0.04	-8509.84	-8510.99
Barisal	-2.68	1.56	1.65	-1.61	1.14	0.35	-1290.3	-1300.5
Dhaka	-2.64	1.63	1.57	-1.55	1.21	0.32	-2034.1	-2050.1
Eastern	-1.79	1.12	0.59	-1.33	1.02	0.04	-770.7	-770.8
Central	-2.11	1.32	0.96	-1.38	1.11	0.12	-732.9	-733.6

Weight for Height								
State	Parameter estimates			Estimates of			Log-likelihood	
	location	scale	shape	mean	sd	skewness	SN	normal
Kerala	-1.59	1.42	0.82	-0.88	1.22	0.09	-1452.04	-1452.91
Uttar Pradesh	0.17	1.56	-1.10	-0.75	1.26	-0.17	-8669.44	-8686.59
Barisal	-1.91	1.53	1.44	-0.91	1.16	0.28	-1304.6	-1313.1
Dhaka	-1.88	1.62	1.30	-0.86	1.26	0.23	-2087.9	-2097.6
Eastern	-1.39	1.38	1.08	-0.58	1.12	0.16	-816.1	-817.8
Central	0.06	1.34	-0.90	-0.66	1.14	-0.11	-745.8	-746.4

Source: Author's own calculation

In order to define a measure of deficit, let us consider the Figure 1. Let τ be the cutoff point defined as following

$$\tau = min\left\{y | \frac{2}{\omega}\phi\left(\frac{y-\xi}{\omega}\right)\Phi\left(\lambda\frac{y-\xi}{\omega}\right) = \phi(y) \gg 0\right\} \quad (6.1)$$

i.e. τ is the smallest value of the random variable Y at which the two curves intersect with a value of probability density greater than zero.. Based on the value of τ, we can define a measure of deficit as

$$\nabla_{def}(\tau) = \int_{-\infty}^{\tau} f(y;\ \xi, \omega, \lambda)dy - \int_{-\infty}^{\tau} \phi(t)dt\ . \quad (6.2)$$

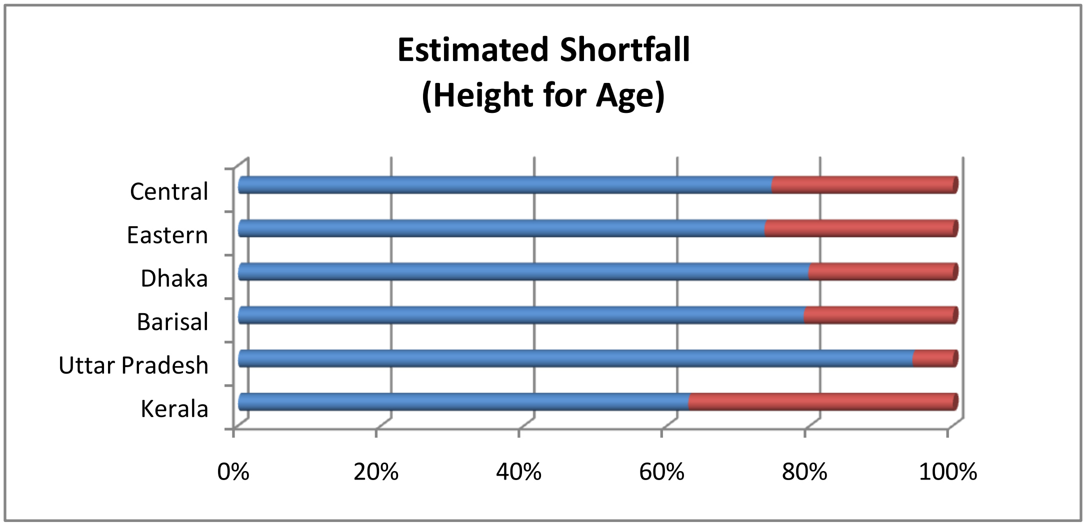
Clearly, $\nabla_{def}(\tau)$ measures the area of shortfall which lies below the ideal nutritional level. Let $\hat{f} = f(y;\ \hat{\xi}, \hat{\omega}, \hat{\lambda})$ be the fitted distribution of the z- score under consideration. Then, an estimate $\hat{\nabla}_{def}(\tau)$ is obtained by using equations (6.1) and (6.2). The following Table 5 provides the estimates of nutritional deficits in selected Indian states using the method described above.

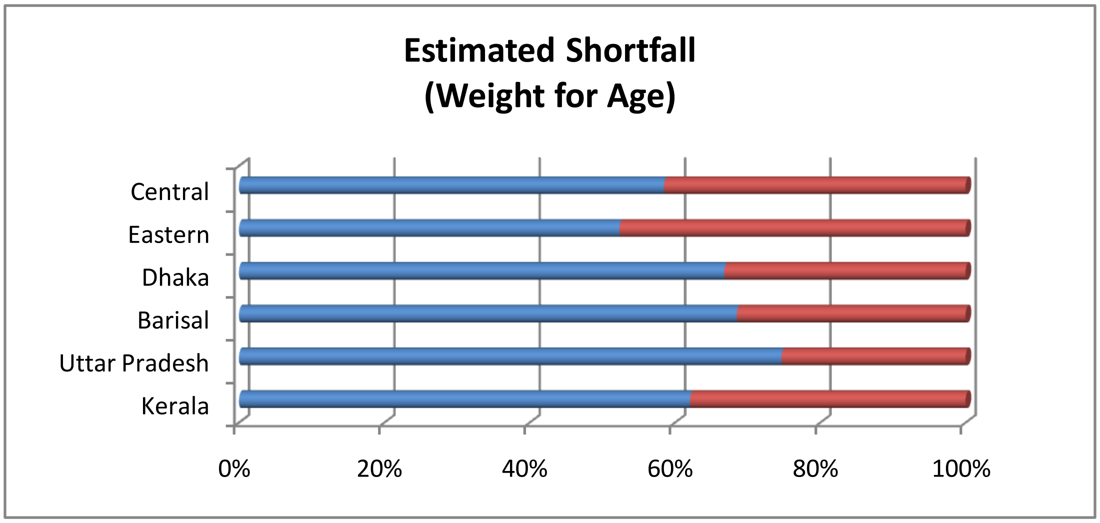
Table 5. Estimated nutritional deficit of various z-scores and percentage below -2sd by selected states/regions of India, 2005-06, Bangladesh and Nepal 2011.

State	Height for Age		Weight for Age		Weight for Height	
	% of $\hat{\nabla}$	% below -2sd	% of $\hat{\nabla}$	% below -2sd	% of $\hat{\nabla}$	% below -2sd
Kerala	38.9	24.5	36.9	22.9	29.9	15.9
Uttar Pradesh	60.2	56.8	56.8	42.4	22.1	14.8
Barisal	54.6	43.2	56.2	38.5	33.6	14.3
Dhaka	53.4	42.6	53.3	35.6	32.0	16.3
Eastern	53.1	39.1	48.7	25.5	20.8	10.3
Central	51.2	38.2	48.9	28.6	19.8	11.4

Source: Author's own calculation

The results obtained in Table 5 are indicative of the fact that there exist comprehensive gaps between the peceived level of undernutrition and the extent of actual nutritional deficit. The weight for age indicator, the only anthropometric index selected to assess progress towards Millennium Development Goals of halving under five undernutrition by 2015, is the worst victim; the nutritional deficit with respect to this indicator is estimated to be 14 to 23 per cent more than the usual estimated prevalence for the selected states/regions. Noticeably, in a progressive state like Kerela in India, it is seen that the estimated deficit exceeds by about 14 per cent with respect to all the chosen indicators, a result that could have been unidentified otherwise. Figure 4 below depicts the estimated shortfall as observed in percentage covered by -2sd cut off of estimated actual deficit. Noticeable extents of shortfalls are seen with respect to weight for age and weight for height indices. Results are far from satisfaction in terms of the extent of coverage.





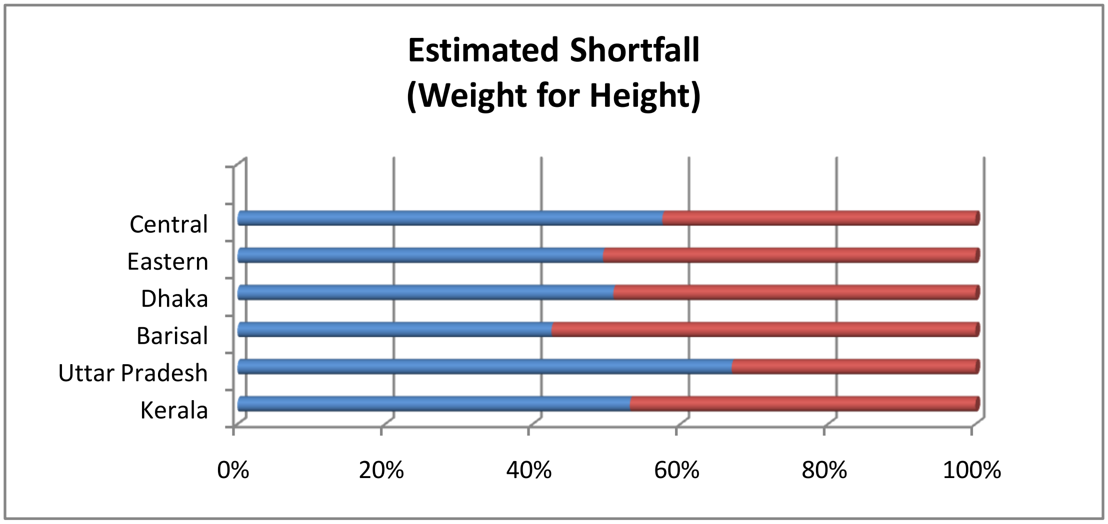


Figure 4. Depicts Percentage covered by -2sd cutoff of estimated actual deficit for various nutritional indicators

7. Discussion and concluding remarks

Despite the popularity and recognized usefulness of nutritional anthropometry in assessments of health and nutrition, there have been many discussions and conflicting recommendations about the cut-off points to be used for estimating the prevalence of undernutrition. Different cut-off points and classification systems have been proposed and used for estimating the prevalence of malnutrition in population surveys; thus the reported rates are often not comparable and sometimes questionable. Researchers observed that in practice, a measure that compares the statistical distributions of the Z-scores for the observed population and the reference, is ideally needed and that would resolve the debate. Mora's method (Mora, 1989) is a one step forward towards this end. However, the assumptions of normality for the distributions have been found to be untenable for the data from developing nations.

In the present work we have moved another step forward from the Mora's method by introducing a location-scale family of skew normal distribution, for the Z-scores of the study populations, which includes normal distribution as a member of this class. In this sense, the method proposed here can be seen as a generalization of Mora's method and can suitably be applied to study undernutrition in the developing countries. We have applied the proposed method to latest NFHS/DHS survey data obtained from the selected states/regions from three countries, India, Bangladesh and Nepal. The results of our analysis suggest that in most of the cases degree of prevalence as measured by the percentage of children below -2sd of reference population, do not substantially differ from the values as reported(IIPS and Macro International, 2007, MOHP et al, 2012, NIPORT et al, 2013), however, there are shortfalls of about 10 – 15 per cent as against their actual deficits. These findings are clearly indicative of the fact that there exist comprehensive gaps in the perceived level of undernutrition and the extent of nutritional deficit for the developing nations. The task of combating the problem of nutrition with various intervention policy measures necessarily be formulated by accounting for such gaps.

In conclusion, we observe that distributional lessons help visualizing the nutritional status at different extremes/tails in a quantifiable manner. While analyzing health inequality among children under age five we should also examine the proximate and distant determinants of malnutrition and how do they vary over the entire support of the distribution. Policy intervention strategies should address the determinants depending on in which part of the distribution the children are located.

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