Exploring Stable Population Concepts from the Perspective of Cohort Change Ratios: Estimating Time to Stability and Intrinsic r*

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Abstract. Cohort Change Ratios (CCRs) have a long history of use in demography. In spite of their history of use, they appear, however, to have been overlooked in regard to a major canon of formal demography, stable population theory. In this paper, CCRs are explored as a tool for examining the idea of a stable population. In comparing the approach using CCRs to the traditional analytical approach, benefits and drawbacks are noted. The paper also introduces an Index of Stability, which is used in a regression model to estimate the number of years before the population in question becomes (approximately) stable. The regression model works reasonably well and, as such, provides something not available in the traditional analytical approach, which is an estimate of the time to (approximate) stability for a given population. Continuing the use of regression analysis, we also find that a regression model works reasonably well in estimating the intrinsic rate of increase from the initial rate of increase. We know that regression models are generally not as satisfying as analytical expressions in regard to describing relationships. It would be much more elegant to express the time to stability in terms of an analytic expression that incorporates the initial stability index (and probably other information about initial conditions) than it is to express the relationship in the form of a regression model. The same can be said about the relationship between the initial rate of increase in a given population and its intrinsic rate of increase. However, we also note that regression analysis has already been successfully employed in conjunction with stable population analysis, to include estimating intrinsic r from the proportional age distribution of a given population, mean generation length from a trial value of the intrinsic rate of increase, and the generation of model life table families and stable populations.

Kewords. Stable Population Index, Hamilton-Perry Method, Intrinsic r, Numerical Solution, Demographic Theory, Weak Ergodicity Theorem

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I. Introduction

Cohort Change Ratios (CCRs) have a long history of use in demography, starting with Hardy and Wyatt (1911). Under the rubric of "Census Survival Ratios," they have been used to estimate adult mortality (Swanson and Tedrow 2013, United Nations 2002) and under the rubric of the "Hamilton-Perry" method, they are used to make population projections (Hamilton and Perry 1962, Smith Tayman and Swanson 2013, Swanson and Tayman 2013, Swanson Schlottmann and Schmidt 2010). However, they appear to have been overlooked in regard to examining the concept of a stable population.[1]

A stable population is a population with an invariable relative age structure and a constant rate of growth. That is, the proportion of people in each age group remains constant over time (Coale 1972, Dublin 1925, Lotka 1907, Preston et al. 2001). When the absolute number of people in each group is also constant over time, a stationary population exists, which is a special case of a stable population in which the growth rate is zero (Preston et al. 2001).

A stable population comes about if a constant set of fertility and mortality rates is applied to an arbitrarily chosen age distribution (Coale 1972, Dublin 1915, Lotka 1907, Preston et al. 2001). That is, if a given population is subjected to constant fertility and mortality rates, it will eventually reach stability. Eventually, when a given population reaches stability, the constant set of fertility and mortality rates produce a constant rate of population change. This constant rate of change is known by several names, but in this paper we use the term intrinsic r (Preston et al. 2001).

In line with observations by Preston et al. (2001), among others, a stable population will results if a constant set of migration rates is included with sets of constant fertility and mortality rates. Since migration, fertility, and mortality rates make up CCRs, this implies that applying a set of constant CCRs to a given population will eventually produce a stable population. This is the main topic of this paper. We also note that there is an intrinsic r (***r***) associated with a stable population produced by applying a constant set of CCRs to a given population.

In pursuing the main topic, the paper also introduces an Index of Stability (***S***), which is used in a regression model to estimate the number of years before the population in question becomes (approximately) stable. As we show later, the regression model works reasonably well and, as such, provides something not available in the traditional analytical approach, which is an estimate of the time to (approximate) stability for a given population. Continuing the use of regression analysis, we also find that a regression model works reasonably well in estimating the intrinsic rate of increase (***r***) from the initial rate of increase. We know that it would be much more elegant to express the time to stability and ***r*** in terms of analytic expressions that incorporate information about initial conditions than it is to express these relationship in the form of regression models. However, we also note that regression analysis has already been successfully employed in conjunction with stable population analysis, to include estimating intrinsic r from the proportional age distribution of a given population (Keyfitz and Flieger 1968: 49, United Nations 1968), mean generation length from a trial value of the intrinsic rate of increase (McCann 1973), and the generation of model life table families and stable populations (Coale and Demeney 1968).

The reminder of this paper is composed of seven sections, in the next one (II), we discuss the CCR method while in Section III we discuss stable population concepts. The CCR approach to the concept of a stable population is discussed in Section IV while Section V describes the estimation of time to stability and Section VI describes the estimation of ***r*** from the initial rate of increase. Section VII concludes the paper with a discussion of the results and ideas for future research.

II. Cohort Change Ratios

Because we use a constant set of CCRs to project a population to stability, we discuss them in conjunction with the Hamilton-Perry method. The Hamilton-Perry Method is a variant of the cohort-component method that has far less intensive input data requirements. Instead of mortality, fertility, migration, and total population data, which are required by the full-blown cohort-component method, the Hamilton-Perry method requires data only from the two most recent censuses (Hamilton and Perry 1962, Smith Tayman and Swanson 2013, Swanson and Tayman 2013, Swanson Schlottmann and Schmidt 2010).

The Hamilton-Perry method moves a population by age (and sex) from time t to time t+k using CCRs computed from data in the two most recent censuses. It consists of two steps. The first uses existing data to develop CCRs and the second applies the CCRs to the cohorts of the launch year population to move them into the future. The second step can be repeated infinitely, with the projected population serving as the launch population for the next projection cycle. The formula for the first step, the development of a CCR is:

$$ {}_{n}CCR_{x,i} = {}_{n}P_{x,i,t} / {}_{n}P_{x-k,i,t-k} \quad [1] $$

where

${}_{n}P_{x,i,t}$ is the population aged x to x+n in area i at the most recent census (t),

${}_{n}P_{x-k,i,t-k}$ is the population aged x-k to x-k+n in area i at the 2nd most recent census (t-k),

k is the number of years between the most recent censuses at time t for area i and the one preceding it for area i at time t-k.

The basic formula for the second step, moving the cohorts of a population into the future is:

$$ {}_{n}P_{x+k,i,t+k} = ({}_{n}CCR_{x,i})*({}_{n}P_{x,i,t}) \quad [2] $$

where

${}_{n}P_{x+k,i,t+k}$ is the population aged x+k to x+k+n in area i at time t+k

${}_{n}CCR_{x,i} = {}_{n}P_{x,i,t} / {}_{n}P_{x-k,i,t-k}$

${}_{n}P_{x,i,t}$ is the population aged x to x+n in area i at the most recent census (t),

k is the number of years between the most recent censuses at time t

for area i and the one preceding it for area i at time t-k.

Given the nature of the CCRs, 10-14 is the youngest age group for which projections can be made if there are 10 years between censuses. To project the population aged 0-4 and 5-9 one can use the Child Woman Ratio (CWR), or more generally a "Child Adult Ratio" (CAR). It does not require any data beyond what is available in the decennial census. There are different ways to develop a CAR (Hamilton and Perry 1962, Smith, Tayman and Swanson 2013: 176-180, Swanson and Tayman 2013, Swanson, Schlottmann and Schmidt 2010). In this paper we take the ratio of the population aged 0-4 to the population aged 20-34 and the ratio of the population aged 5-9 to the population aged 25-39. Here are the CAR equations for projecting the population aged 0-4 and 5-9, respectively.

Population 0-4: ${}_{5}P_{0,t+k} = ({}_{5}P_{0,t} / {}_{20}P_{15,t}) * ({}_{20}P_{15,t+k})$ [3]

Population 5-9: ${}_{5}P_{5,t+k} = ({}_{5}P_{5,t} / {}_{25}P_{15,t}) * ({}_{25}P_{15,t+k})$ [4]

where

P = population,

t is the year of the most recent census

and t+k is the estimation year

While there are other “adult” age groups that could be used to define CAR, we prefer the preceding for purposes of this paper because the definitions shown in the two preceding equations are designed for a population in which fertility is at or below replacement, (i.e., the TFR is less than 2.1 or so), which correlates with the fact that first births tend to be both postponed and low in number.

Projections of the oldest open-ended age group differ slightly from the CCR projections for the age groups beyond age 10 up to the oldest open-ended age group. If, for example, the final closed age group is 80-84, with 85+ as the terminal open-ended age group, then calculations for the ${}_{\infty}CCR_{85,i,t}$ require the summation of the three oldest age groups to get the population age 75+ at time t-k:

$$ {}_{\infty}CCR_{75,i,t} = {}_{\infty}P_{85,i,t} \; / \; {}_{\infty}P_{75,i,t-k} \qquad [5]$$

The formula for projecting the population 85+ of area i for the year t+k is:

$$ {}_{\infty}P_{85,imt+k} = ({}_{\infty}CCR_{75,i,t})^{*} \, ({}_{\infty}P_{75,i,t}) \qquad [6]$$

Table 1 provides an example of the Hamilton-Perry Method for the state of Alaska. It uses the country’s 2000 census data and 2010 estimates by age to generate a 2020 population projection of the population by age. Since the population data are ten years apart for Alaska with a final open-ended age group of 85+, the conventions described above are used in terms of the CCRs, CAR, and the projection of age group 85+. Important to the subsequent discussion are the CCRs developed for the 2000-2010 period.

(Table 1 About Here)

Table 1 shows that launching from a population of 710,231 in 2010, the Hamilton-Perry Method generates a 2020 population of 795,728. This projection corresponds to the increase in population between 2000 (626,932) and 2010 (710,231). This increase largely reflects Alaska's net in-migration and relatively young population. Since this touches on the implicit recognition of the components of population change in the Hamilton-Perry projection for Alaska, it is worthwhile to note here the Hamilton Perry method can be described in terms of these components. That is, the Hamilton-Perry Method can be expressed in terms of the fundamental demographic equation.[2] Since the fundamental equation is:

$$P_{i,t+k} = P_{i,t} + B_i - D_i + I_i - O_i \qquad [7]$$

where

$P_{i,t}$ = Population of area i at time t (e.g., the launch date)

$P_{i,t+k}$ = Population of area i at time t+k (e.g., the projection target date)

B_i = Births in area i between time t and t+k

D_i = Deaths in area i between time t and t+k

I_i = In-migrants in area i between time t and t+k (including international and domestic sources)

O_i = Out-migrants in area i between time t and t+k (including international and domestic destinations)

Equation [1] can be expressed as

$$_nCCR_{x,i} = {}_nP_{x,i,t} / {}_nP_{x-k,i,t-k} \qquad [8]$$

since

$${}_{n}CCR_{x,i} = ({}_{n}P_{x-k,i,t-k} + B_i - D_i + I_i - O_i)/({}_{n}P_{x-k,i,t-k}) \qquad [8.a]$$

while equation [2] can be expressed as

$${}_{n}P_{x+k,i,t+k} = ({}_{n}CCR_{x,i})*({}_{n}P_{x,i,t}) \qquad [9]$$

since

$${}_{n}P_{x+k,i,t+k} = (({}_{n}P_{x-k,i,t-k} + B_i - D_i + I_i - O_i)/({}_{n}P_{x-k,i,t-k}))*({}_{n}P_{x,i,t}) \qquad [9.a]$$

where x+k >= 10 then

$${}_{n}CCR_{x,i} = ({}_{n}P_{x-k,i,t-k} - D_i + I_i - O_i)/({}_{n}P_{x-k,i,t-k}) \qquad [9.b]$$

and since $N_i = I_i - O_i$

$${}_{n}CCR_{x,i} = ({}_{n}P_{x-k,i,t-k} - D_i + N_i)/({}_{n}P_{x-k,i,t-k}) \qquad [9.c]$$

where x+k >= 10

These equations clearly reveal that the Hamilton-Perry Method expresses the individual components of change (birth, deaths, and migration) in terms of cohort change ratios and incorporates these components of change in the projections made from it. Note that the fundamental equation can be generalized to include age groups (as well as sex, race, and ethnicity).

III. A Stable Population: The Traditional Approach

As noted earlier, a stable population is a population with an invariable relative age structure and a constant rate of growth. That is, the proportion of people in each age group remains constant over time and the population as a whole has a constant rate of increase (Coale 1972, Dublin 1925, Lotka 1907, Preston et al. 2001).

An important feature of the stable population model is that over time a population "forgets" its past age distribution when it is subject to constant vital rates regarding the components of change (Coale 1972, Cohen 1979, Preston et al. 2001). This property is known as weak ergodicity (Cohen 1979).

Alfred J. Lotka is generally credited with formulating the idea of a stable population and exploring many of its important features, including the finding that in the absence of migration, a population subject to constant fertility and mortality rates would eventually have a constant rate of natural increase (Dublin 1925, Lotka 1907). Continuing the analytical tradition established by Lotka, many researchers have examined the idea of a stable population and refined its underlying theory and extended its applications (Alho and Spencer 2005, Arthur and Vaupel 1984, Bacaër 2011, Bennett and Horuchi 1984, Caswell 2001, Coale 1972, Cohen 1979, Kim and Sykes 1976, Le Bras 2008, Pollard et al. 1974, Popoff and Judson 2004, Preston et al. 2001, Preston and Coale 1982, Rogers 1985, Schoen 1985, United Nations 1968). Most of this research has, however, been confined to examining a population not affected by migration. However, this is an un-necessarily restrictive assumption (Preston et al. 2001). Nonetheless, other than the simple migration rates employed by Rogers (1985, 1995) and subsequent investigations of more refined model migration schedules (Rogers et al. 1986), this restriction appears to remain a governing force in the examination of stable population ideas.

Another “unnecessarily restrictive” assumption that has governed much of the work on stable populations is defined by the so-called “two-sex” problem (Pollak 1986, 1990, Preston and Coale 1982). In this problem (which evidently stems from Lotka’s 1907 formulation of a stable population), only one sex (virtually always women) was examined in the context of a stable population because of problems reconciling the numbers of births resulting from including both sexes. However, as Preston et al. (2001)

show a “female-dominant” approach to fertility offers a convenient way around this problem, one that has been employed in different ways by others (Barclay, 1958: 216-222; Keyfitz and Flieger, 1968).

Although Preston et al. (2001) point out that the assumption of no migration is un-necessarily restrictive, stable population theory has largely been examined using this restriction. It also has largely been examined in terms of a single sex due to the so-called “two-sex” problem, which Preston et al. (2001) also argue is un-necessarily restrictive.

The Lotka Integral Equation as given by Preston et al. (2001) is

$$B(t) = \int_0^t N(a,t)m(a)da + G(t) \quad [10]$$

where

B(t) = number of births at time t

N(a,t) = number of persons aged a at time t

m(a) = rate of bearing female children for women aged a

G(t) = births to women alive at time 0

As Preston et al. (2001) observe, the N(a,t) function for women born after time 0 can be expressed in terms of the number of births into their cohort and the probability of surviving to age a, p(a):

$$N(a,t) = B(t-a)*p(a) \quad [11]$$

where $t>0$

Making this substitution into the preceding equation yields

$$B(t) = \int_0^t B(t-a)*p(a)* m(a)da + G(t) \quad [12]$$

And since the value of G(t) goes to zero over time (e.g., in about 50 years), the birth sequence can be expressed as

$$B(t) = \int_0^t B(t-a)*p(a)*m(a)da \qquad [13]$$

where t> 50

The preceding Equation can be solved when an expression for B(t) is substituted into its left and right hand sides. Lotka showed that an exponential birth series would do this. Let $B(t) = B*e^{pt}$

Then

$$B*e^{pt} = \int_0^t B*e^{p(t-a)}*p(a)*m(a)da \qquad [14]$$

where t> 50

and cancelling the common term, $B*e^{pt}$ from both sides yields

$$1 = \int_0^t B*e^{pa}*p(a)*m(a)da \qquad [15]$$

The ideas expressed in equations [10 to [15] are usually used to estimate "intrinsic r," the rate of population increase when a given population in question attains stability Barclay (1958), Keyfitz and Flieger (1968) and Preston et al. (2001).

III. A Stable Population: The CCR Approach

The CCR approach simply takes the cohort change ratios found at a current point in time and holds them constant until the population reaches stability. To determine when a population has reached stability, the well-known "Index of Dissimilarity" is employed as an "Index of Stability" (***S***).[3] The index is defined as:

$$\boldsymbol{S} = \{0.5* \sum | ({}_np_x/\sum {}_nP_x)_{t+y} - ({}_np_x/\sum {}_nP_x)_t |\}. \qquad [16]$$

where

y = number of years between census counts/projection cycles
x = age
n = width of the age group (in years)
t = year

S compares the relative age distribution at one point in time (t+y) with the relative age distribution at the preceding point in time (t) and measures the percentage that one age distribution would have to be re-allocated to match the other. ***S*** ranges from 0 to one (1); a score of zero is means that there is no allocation error, and a score of one (1) means that the maximum allocation error exists. A score of one (1) can be interpreted in several ways, but a common interpretation is that half of the numbers by age in one population would have to be re-allocated in order to match the distribution of the numbers by age in the comparison population. Since we are dealing with the same population at viewed at two successive points in time, this leads to viewing a score of one (1) as an indication that one half of the numbers by age at time t would have to be reallocated to match the numbers by age of the same population at the preceding point in time.[3]

S exploits the idea that when a population is stable, the sum of the differences between the relative size of corresponding age groups at time t+y and time t is zero. Thus, at a point time when the sum of the differences across all of the corresponding age groups is zero at that point in time and the preceding point in time (or very nearly so), the population has reached stability. The advantage of using the Index of Dissimilarity as ***S*** is that it provides ***S*** with a bounded measure (between 0 and 1) and has a clear interpretation. This index could, of course, be used in conjunction with the traditional approach, but it does not appear in the literature in regard to measuring population stability. With ***S***, one has a potential tool for examining the length of time to stability for given population.

The examination of the CCR approach to the idea of a stable population starts by using the case of Alaska. The CCRs (from the 2000-2010 period) are held constant from the launch year (2010) to a year where ***S*** = 0 (relative to the preceding year in the projection cycle). This occurs at the year 2470. Table 2 displays this by showing the information at for the 2000-2010 launch period and the information at the period where stability is reached, 2370-2380. ***S*** = .10573 at the launch year of 2010; by 2380, ***S*** = 0.0000.

(Table 2 About Here)

Figure 1 provides the change in ***S*** from 2010 to 2470. As it shows, the path to stability is monotonic but not linear. It initially declines rapidly to the point where S is approximately equal to .01, but the change in ***S*** slows substantially around 2120. From there to 2470, ***S*** moves incrementally to zero.

(Figure 1 About Here)

Figures 2 and 3 show the age distribution of Alaska in 2010 and in 2470, when it reaches stability.

(Figures 2 and 3 About Here)

As another example, consider the United States. As was the case with Alaska, the projection is launched with CCRs taken over the 2000-2010 period that are held constant from the launch year to a year where ***S*** = 0 (relative to the preceding year in the projection cycle). Stability occurs at the year 2380. Table 3 displays this by showing the information at the launch period, 2000-2010, and the information at the period where stability is reached, 2330-2340. $\boldsymbol{S}$ = .0565 at the launch year of 2010; by 2380, $\boldsymbol{S}$ = 0.0000

(Table 3 About Here)

Figure 4 provides the change in $\boldsymbol{S}$ from 2010 to 2340 for the United States. Unlike Alaska, the path to stability is neither monotonic nor linear: It initially increases, "bounces around" a bit, and then decreases substantially before its decrease slows considerably, which starts around 2160. From 2160 to 2340, $\boldsymbol{S}$ moves incrementally to zero. Figure 5 shows the graph of ln($\boldsymbol{S}$) relative to time

(Figures 4 and 5 About Here)

Figures 6 and 7 show the age distribution of the U.S. in 2000 and in 2340, when it reaches stability

(Figures 6 and 7 About Here)

The final case study population is Whitman County, Washington. This population is of interest not only because it is growing but because it is heavily impacted by a "special population, namely students enrolled at Washington State University. In 2010, the total population of Whitman County was about 45,000. Students at Washington State University make up about half of this number. This can be seen in Figure 10.

As was the case with Alaska and the United States, the projection is launched with CCRs taken over the 2000-2010 period, which are held constant from the launch year to a year where $\boldsymbol{S}$ = 0 (relative to the preceding year in the projection cycle). This occurs at the year 2290. Table 4 displays this by showing the information at the launch period, 2000-2010, and the information at the period where stability is reached, 2330-2340. $\boldsymbol{S}$ = 0.0798 at the launch year of 2010; by 2290, $\boldsymbol{S}$ = 0.0000

(Table 4 About Here)

Figure 8 provides the change in $\boldsymbol{S}$ from 2010 to 2290 for Whitman County. As was the case for the United States, the path to stability is neither monotonic nor linear: It initially increases, "bounces around" a bit, and then decreases substantially before its decrease slows considerably, which starts around 2160. From 2160 to 2290, $\boldsymbol{S}$ moves incrementally to zero. Figure 9 shows the graph of ln($\boldsymbol{S}$) relative to time

(Figures 8 and 9 About Here)

Figures 10 and 11 show the age distribution of Whitman County in 2000 and in 2290, when it reaches stability

(Figures 10 and 11 About Here)

To examine the question of weak ergodicity, the 2000-2010 CCRs for the USA are applied to Whitman County, which reaches stability in 2420 using these CCRs. Table 5 contains the data while Figure 12 shows the age distribution of Whitman County in 2240 when it reaches stability using the US CCRs. In comparing the age distribution found in Figure 12 to that of the US (at stability) in 2340, it is clear that they are very similar, if not identical. To test this more rigorously, the differences were calculated and found to be essentially zero at each age group. In addition, the intrinsic growth rate of .00475 matches that of the US when it reaches stability. This confirms the idea that using CCRs to generate stable populations is consistent with ergodicity theory, at least in its weak form (Cohen 1979). The test results are in Table 6.

(Figure 12 About Here)

(Table 6 About Here)

IV. Time to Stability

The analytic approach to a stable population does not provide a means to estimate the time required before a given population achieves stability. In looking at a scatter plot of the initial Stability Score and the time to Stability, it is apparent that a positive linear relationship exists (see Figure 13). Thus, it was natural to look toward regression analysis as a way to estimate time to Stability from the initial value of $\boldsymbol{S}$. Thus, a simple bivariate regression model was constructed using a sample of 18 U.S. States used in a different study (Swanson and Hough 2012). These states are shown in Exhibit 1.

(Exhibit 1 About Here)

(Figure 13 About Here)

The regression model was constructed using one independent variable, the initial value of $\boldsymbol{S}$. The Dependent variable is time (in years) to "stability." Of course, there is more information available for a population at its time of launch (e.g., proportion of the population under 20 years of age, the initial rate of population change) that could be examined as potential independent variables in a multiple regression model. However, it is seems obvious that since the larger an $\boldsymbol{S}$ score, the farther a population is from stability, the initial $\boldsymbol{S}$ score should serve as the starting point in a regression model. That is, the hypothesis is that there is a positive relationship between initial $\boldsymbol{S}$ score and time to stability.

Population stability is measured "approximately" by selecting the time to stability defined as when $\boldsymbol{S}$ =0.01. That is, when only one percent of age distribution the population at the preceding year needs to be re-allocated to match the age distribution of the population at the subsequent year. ***S*** =0.01 was selected because an examination of the scatter plots for Alaska, The United States, and Whitman County revealed that a long “tail” exists in going from ***S*** =0.01 to ***S*** =0.00 (see, e.g. Figures 1, 5, or 9). Because U.S. states are used, there are ten years between these two points in time. Figure 13 shows the relationship between the Initial ***S*** score and the time to ***S*** =0.01.

The NCSS statistical system was used to build the regression model, an overview of which is given below. The input data used to build the regression model are found in Table 7.

(YEARS TO ***S*** =0.01) = 31.42 + (861.53*INITITAL_***S***_SCORE)

(p= .047) (p = .0011)

$r^2 = .495$

Both the intercept and the partial regression coefficient for the initial ***S*** score is statistically significant ($\alpha = 0.05$) and that the coefficient of determination suggests that the model explains 50 percent of the variation in years to approximate stability (***S*** =0.01).

(Table 7 About Here)

To get an idea of the accuracy of the model shown in the regression equation, it was used to estimate time to ***S*** =0.01 for the case study populations, Alaska, the United States, and Whitman County, Washington. Table 8 provides the results of this examination.

(Table 8 About Here)

The estimates for the US and Whitman County are reasonably accurate, with error of -8 and -19 years respectively and the estimate for Alaska is very accurate in that the time to approximate stability at $\boldsymbol{S}$ =0.01 is estimated as 123 years and the actual number of years to $\boldsymbol{S}$ =0.01 is 120.

VI. Estimating Intrinsic r from the initial rate of population increase

A range of methods exist for estimating intrinsic r (Barclay 1958: 216-222, Coale 1957, 1972, Dublin 1925, Keyfitz and Flieger 1968; Lotka 1907, McCann 1973, Pressat 2009: 318-328, Preston et al. (2001:138-170, United Nations 1968), but we not aware of the direct use of regression analysis using the initial rate of increase in a given population for this purpose.[5] We note that analytic methods are preferable when relationships are understood. However, as Barclay (1958: 216) observes the determination of a non-stationary population is a complex task and the literature does not reveal a direct relationship between the initial rate of increase in a given population to its intrinsic rate of increase (Barclay 1958, Coale 1957, 1972, Dublin 1925, Keyfitz and Flieger 1968, Lotka 1907, McCann 1973, Pressat 2009, Preston et a. 2001) As an initial exploration of this relationship, and given the positive results yielded from employing regression to estimate the time to stability for a given population, we, therefore, employ regression analysis.

As a first step, we use data on 67 countries found in Keyfitz and Flieger (1968) in a “proof of concept” test. These 67 cases represent are the most recent entries for national and ethnic populations in Keyfitz and Flieger (1968); they also were used by McCann (1973) in constructing a quadratic regression model to estimate mean generation length, which he then employed to estimate intrinsic r in conjunction with the natural logarithm of the net reproduction rate. The independent variable is the natural rate of increase, which Keyfitz and Flieger (1968) found by subtracting the crude death rate from the crude death rate for these 67 populations. The dependent variable is the intrinsic rate of increase found by Keyfitz and Flieger for these same 67 populations. As an example, the initial rate of increase used for Costa Rica in 1963 is 41.31 (.4131) while its intrinsic rate of increase is 41.5200 (.415200) (Keyfitz and Flieger 1968: 96). The complete set of data is found in Table 9 while Figure 14 provides a scatter plot between the initial rate of increase (***r***) and the intrinsic rate of increase (***r'***).

(Table 9 About Here)

(Figure 14 About Here)

It is clear from Figure 14 that a positive linear relationship exists between ***r*** and ***r'***. The regression model constructed from the data in Table 9 using the NCSS Statistical System is:

$$r' = -1.1719 + 1.0532*r$$

(p= .0222)   (p <.0001)

$r^2 = .8992$

The results strongly support the idea that ***r'*** can be estimated from ***r*** using linear regression. The coefficient of determination is very high and the slope coefficient is statistically significant. Given this, we now turn our attention to the same 18 county data set used to generate the regression model used to estimate time to stability from the score of the initial stability index (***S***). The counties are named in Exhibit 1 while the values of ***r*** and ***r'*** for these 18 counties are provided in Table 10.

(Table 10 About Here)

The scatter plot between ***r*** (x axis) and ***r'*** (y axis) for these 18 counties found in Figure 15 shows a positive linear relationship between these two variables and is consistent with what was observed in Figure 14.

(Figure 15 About Here)

The regression model constructed from the data in Table 10 using the NCSS Statistical System is:

$$r' = -0.0036 + 0.9561*r$$

(p= .0004)   (p < .0001)

$r^2 = .9302$

As was the case with the "proof of concept" test using the data from Keyfitz and Flieger (1968), the results for the model constructed using the data for the 18 show that that ***r'*** can be estimated from ***r*** using linear regression. The coefficient of determination is very high and both the intercept terms and the slope coefficient are statistically significant. To get an idea of the accuracy of the model shown in the regression equation, it was used to estimate ***r'*** from *r* for the three case study populations, Alaska, the United States, and Whitman County, Washington. Table 11 provides the results of this examination.

(Table 11 About Here)

The estimate for Whitman County is quite accurate with an error of -.0038 (-5.02 percent) while the estimates for Alaska and the United States are reasonably accurate, with errors of 0.0004 (4.94 percent) and 0.0005 (10.8 percent), respectively.

VII. Conclusion

Cohort Change Ratios (CCRs) appear to us to be useful as a tool for examining the idea of a stable population, given the informal and non-rigorous examination found in this paper. Benefits of the CCR approach include the ability to easily deal with both sexes and all of the components of change, including migration. A drawback of the CCR approach is that one cannot easily assess the effect of each component of change since they are all effectively rolled into CCRs.

While we have not provided mathematical proofs, we believe that the numerical results support the idea that applying cohort change ratios to a given age distribution will result in a stable population and that this result is consistent with the weak ergodicity theorem. In addition, there are three by-products of this paper that we believe are useful: (1) the Index of Stability (***S***); (2) the use of S to estimate time to stability; and (3) estimating intrinsic r (***r'***) directly from a given population's initial rate of increase (***r***). As noted earlier, we were unable to find anything similar to ***S*** in the stable population literature and, in particular, it use to define stability.[6] Calling upon the Index of Dissimilarity for this purpose appears to be a natural use for it and the data suggest that when ***S***=0, a population has reached stability. The use of ***S*** in estimating time to stability via a regression model also appears to us to be useful. The third by-product is the use of regression to estimate intrinsic r (***r'***) from the initial rate of increase for a given population (***r***). Again, the results suggest that intrinsic r can be estimated using regression and as was the case for ***S*** and its use in estimating time to stability, we are not able to find anything in the stable population literature that this had been done before.

In applying the US CCRs to Whitman County, when this population reached stability, its age distribution was the same as that found for the US when the latter reached stability. That is, as suggested by formal stable population theory, this case study shows that when a constant set of rates is applied to a given population, the initial age distribution is "forgotten" as the population becomes stable. Again, we point out that this is consistent with the weak ergodicity theorem.

The CCR approach appears to be sufficiently useful to warrant further investigation. In this studies, it appears it would be useful to graph The Stability Index over the time it takes a given population to reach stability. The graphs presented here for the three case studies suggest a non-monotonic and non-linear path and similar results (not shown here) were found for the sample of 18 states. The work with the 18 states suggests that regression models provide a way to estimate both time to stability and intrinsic r. These findings suggest that there are relationships within the CCRs and the initial set of conditions in the launch population and the population preceding it (from which the CCRs are constructed along with initial *S* and the initial rate of increase, ***r***) that hold the key to developing analytic expressions for the relationships between these initial conditions, on the one hand, and the time to stability and ***r'***, on the other.[7]

It may be the case that both the regression model used to estimate time to stability and the model used to estimate ***r'*** are accurate only within "families" of population dynamics. Here, the types of dynamics come to mind that are analogous to the Regional Model Life Tables and Stable Populations developed by Coale and Demeny (1996). If this is the case, then the different families would need to be identified and regression models specific to each family would need to be constructed using data from the populations with each family.

Another area for research is the use of CCRs in conjunction with ideas promulgated by Keyfitz (1974) for examining stable processes across two (or more) interacting populations. Because it can deal with both sexes and migration quite handily, the CCR approach may be more tractable in regard to examining the path to stability in such populations.

In conclusion, we know that regression models are generally not as satisfying as analytical expressions in regard to describing relationships. It would be much more elegant to express the time to stability in terms of an analytic expression that incorporates the initial stability index (and probably other information about initial conditions) than it is to express the relationship in the form of a regression model.[8] The same can be said about the relationship between the initial rate of increase in a given population and its intrinsic rate of increase. However, we also note that regression analysis has already been successfully employed in conjunction with stable population analysis, to include the Bourgeois-Pichat method for estimating intrinsic r from the proportional age distribution of a given population (Keyfitz and Flieger 168:49, United Nations 1968), McCann's (1973) method for estimating mean generation length from a trial value of the intrinsic rate of increase, and the generation of model life table families and from them, stable populations (Coale and Demeny 1968).

Endnotes

1. While not an exception to this statement, Sprague (2012) employs a Leslie Matrix in conjunction with cohort change ratios, which could be used to move an initial population to stability. In unpublished work, Swanson (2013) also has developed a Leslie Matrix approach that implements the cohort change ratio approach using a macro written in Visual Basic Applications for excel. The program and excel file is for 16 5-year age groups (0-4, 5-9,…,75+) and a five-year projection cycle. The file includes documentation and instructions for running the macro. It is available on request from David Swanson (dswanson@ucr.edu). Part of this paper is taken form Swanson and Tedrow (2013).
2. We thought it useful to show that the moving a population through time using cohort change ratios is algebraically equivalent to the fundamental demographic equation for two reasons. First, as noted by Land (1986) any quantitative approach to forecasting is constrained to satisfy various mathematical identities, and a demographic approach

should ideally satisfy demographic accounting identities, which is summarized in the fundamental demographic equation. The second reason is based on the argument by Vaupel and Yaushim (1985) that a demographic forecasting method needs to be consistent with the fundamental demographic equation in order to minimize the potential errors associated with hidden heterogeneity.

3. Often, the Index of Dissimilarity is expressed as a percentage, whereby the formula shown in equation [16] is multiplied by 100.

4. We note that according to stable population theory, once stability is achieved, it is maintained, such that the relative size of the age groups of the population in question remain constant throughout time thereafter given that the components of population change that led to stability also remain constant. We assume that this is the case in that once ***S*** reaches zero, it will remain at zero given that the CCRs that led to ***S***=0 also remain constant.

5. While it appears that regression analysis has not been used to estimate intrinsic r from an initial r, Bourgeois-Pichat employed it to estimate intrinsic r from the proportional age distribution of a given population (see Keyfitz and Flieger 1968: 40).

6. Keyfitz and Flieger (1968: 23 and 24-41) show a "dissimilarity" score between a current population age distribution and the age distribution for the corresponding stable population. The index is the sum of positive differences between the two distributions. This index is only one simple step from the Index of Dis-similarity. However, even so it is neither employed by Keyfitz and Flieger to define a stable population nor used to estimate time to stability.

7. As one example of the "initial conditions" information available at launch, the Child Adult Ratio used to project children age 0-4 and 5-9 in the CCR approach is very similar to the "Replacement Index," described by Barclay (1958: 215-216) as an approximation of the net reproduction rate.

8. In regard to the usefulness of empirical findings, we note that in discussing the exploration of Kim and Sykes (1976) in regard to stable population concepts, Cohen (1979: 286) observed that their numerical experiments uncovered empirical regularities that invite theoretical explanation.

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Table 1. A Hamilton-Perry Population Projection for Alaska: Base Year Data (2000-2010), Launch Year(2010) and Target Year 2020)

	2000 POPULATION	2000 PROPORTION BY AGE	2010 POPULATION	2010 PROPORTION BY AGE	2000-2010 CCR	ABS Difference	PROJECTED 2020
Total Population: 0 to 4 years	47,591	0.0759	53,996	0.0760	0.34274	0.0001	58,802
Total Population: 5 to 9 years	53,771	0.0858	50,887	0.0716	0.34162	0.0141	62,124
Total Population: 10 to 14 years	56,661	0.0904	50,816	0.0715	1.06776	0.0188	57,655
Total Population: 15 to 19 years	50,094	0.0799	52,141	0.0734	0.96969	0.0065	49,344
Total Population: 20 to 24 years	39,892	0.0636	54,419	0.0766	0.96043	0.0130	48,805
Total Population: 25 to 29 years	42,987	0.0686	55,419	0.0780	1.10630	0.0095	57,684
Total Population: 30 to 34 years	46,486	0.0741	47,706	0.0672	1.19588	0.0070	65,079
Total Population: 35 to 39 years	55,723	0.0889	45,833	0.0645	1.06621	0.0243	59,088
Total Population: 40 to 44 years	58,326	0.0930	47,141	0.0664	1.01409	0.0267	48,378
Total Population: 45 to 49 years	53,515	0.0854	54,726	0.0771	0.98211	0.0083	45,013
Total Population: 50 to 54 years	41,437	0.0661	56,300	0.0793	0.96526	0.0132	45,504
Total Population: 55 to 59 years	27,423	0.0437	49,971	0.0704	0.93378	0.0266	51,102
Total Population: 60 to 64 years	17,327	0.0276	35,938	0.0506	0.86729	0.0230	48,829
Total Population: 65 to 69 years	12,626	0.0201	22,202	0.0313	0.80961	0.0111	40,457
Total Population: 70 to 74 years	9,881	0.0158	13,148	0.0185	0.75882	0.0028	27,270
Total Population: 75 to 79 years	6,863	0.0109	8,892	0.0125	0.70426	0.0016	15,636
Total Population: 80 to 84 years	3,695	0.0059	5,985	0.0084	0.60571	0.0025	7,964
Total Population: 85 years and over	2,634	0.0042	4,711	0.0066	0.35711	0.0024	6,995
Total Population	626,932	1.0000	710,231	1.0000			795,728

Table 2. The Population of Alaska at start (2000-10) and at achieving Stability (2460-70)

	2000 POPULATION	2000 PROPORTION BY AGE	2010 POPULATION	2010 PROPORTION BY AGE	2000-2010 CCR	ABS Difference	PROJECTED 2460	2460 PROPORTION BY AGE	PROJECTED 2470	2470 PROPORTION BY AGE	ABS Difference	
Total Population: 0 to 4 years	47,591	0.0759	53,996	0.0760	0.34274	0.0001	731,574	0.0695	775,255	0.0695	0.0000	Total Population: 0 to 4 years
Total Population: 5 to 9 years	53,771	0.0858	50,887	0.0716	0.34162	0.0141	745,864	0.0708	790,395	0.0708	0.0000	Total Population: 5 to 9 years
Total Population: 10 to 14 years	56,661	0.0904	50,816	0.0715	1.06776	0.0188	737,139	0.0700	781,149	0.0700	0.0000	Total Population: 10 to 14 years
Total Population: 15 to 19 years	50,094	0.0799	52,141	0.0734	0.96969	0.0065	682,501	0.0648	723,254	0.0648	0.0000	Total Population: 15 to 19 years
Total Population: 20 to 24 years	39,892	0.0636	54,419	0.0766	0.96043	0.0130	668,076	0.0635	707,971	0.0635	0.0000	Total Population: 20 to 24 years
Total Population: 25 to 29 years	42,987	0.0686	55,419	0.0780	1.10630	0.0095	712,506	0.0677	755,051	0.0677	0.0000	Total Population: 25 to 29 years
Total Population: 30 to 34 years	46,486	0.0741	47,706	0.0672	1.19588	0.0070	753,928	0.0716	798,937	0.0716	0.0000	Total Population: 30 to 34 years
Total Population: 35 to 39 years	55,723	0.0889	45,833	0.0645	1.06621	0.0243	716,882	0.0681	759,679	0.0681	0.0000	Total Population: 35 to 39 years
Total Population: 40 to 44 years	58,326	0.0930	47,141	0.0664	1.01409	0.0267	721,471	0.0685	764,552	0.0685	0.0000	Total Population: 40 to 44 years
Total Population: 45 to 49 years	53,515	0.0854	54,726	0.0771	0.98211	0.0083	664,376	0.0631	704,056	0.0631	0.0000	Total Population: 45 to 49 years
Total Population: 50 to 54 years	41,437	0.0661	56,300	0.0793	0.96526	0.0132	657,167	0.0624	696,410	0.0624	0.0000	Total Population: 50 to 54 years
Total Population: 55 to 59 years	27,423	0.0437	49,971	0.0704	0.93378	0.0266	585,431	0.0556	620,379	0.0556	0.0000	Total Population: 55 to 59 years
Total Population: 60 to 64 years	17,327	0.0276	35,938	0.0506	0.86729	0.0230	537,852	0.0511	569,956	0.0511	0.0000	Total Population: 60 to 64 years
Total Population: 65 to 69 years	12,626	0.0201	22,202	0.0313	0.80961	0.0111	447,266	0.0425	473,972	0.0425	0.0000	Total Population: 65 to 69 years
Total Population: 70 to 74 years	9,881	0.0158	13,148	0.0185	0.75882	0.0028	385,125	0.0366	408,131	0.0366	0.0000	Total Population: 70 to 74 years
Total Population: 75 to 79 years	6,863	0.0109	8,892	0.0125	0.70426	0.0016	297,239	0.0282	314,992	0.0282	0.0000	Total Population: 75 to 79 years
Total Population: 80 to 84 years	3,695	0.0059	5,985	0.0084	0.60571	0.0025	220,134	0.0209	233,273	0.0209	0.0000	Total Population: 80 to 84 years
Total Population: 85 years and over	2,634	0.0042	4,711	0.0066	0.35711	0.0024	262,969	0.0250	278,668	0.0250	0.0000	Total Population: 85 years and over
Total Poulation	626,932	1.0000	710,231	1.0000		0.2115	10,527,502	1.0000	11,156,080	1.0000	0.0000	Total Population

Table 3. The Population of the United States at start (2000-10) and at achieving Stability (2330-40)

	2000 POPULATION	2000 PROPORTION BY AGE	2010 POPULATION	2010 PROPORTION BY AGE	2000-2010 CCR	ABS Difference	PROJECTED 2020	PROJECTED 2330	2330 PROPORTION BY AGE	PROJECTED 2340	2340 PROPORTION BY AGE	ABS Difference	
Total Population: 0 to 4 years	19,175,798	0.0681	20,201,362	0.0654	0.32245	0.0027	21,754,486	100,481,615	0.0627	105,625,175	0.0627	0.0000	Total Population: 0 to 4 years
Total Population: 5 to 9 years	20,549,505	0.0730	20,348,657	0.0659	0.33226	0.0071	22,492,130	103,926,932	0.0648	109,244,576	0.0648	0.0000	Total Population: 5 to 9 years
Total Population: 10 to 14 years	20,528,072	0.0729	20,677,194	0.0670	1.07830	0.0060	21,783,056	103,072,841	0.0643	108,348,964	0.0643	0.0000	Total Population: 10 to 14 years
Total Population: 15 to 19 years	20,219,890	0.0718	22,040,343	0.0714	1.07255	0.0005	21,824,924	106,034,347	0.0662	111,466,686	0.0662	0.0000	Total Population: 15 to 19 years
Total Population: 20 to 24 years	18,964,001	0.0674	21,585,999	0.0699	1.05154	0.0025	21,742,806	103,102,009	0.0643	108,384,764	0.0643	0.0000	Total Population: 20 to 24 years
Total Population: 25 to 29 years	19,381,336	0.0689	21,101,849	0.0683	1.04362	0.0005	23,001,707	105,270,843	0.0657	110,659,394	0.0657	0.0000	Total Population: 25 to 29 years
Total Population: 30 to 34 years	20,510,388	0.0729	19,962,099	0.0647	1.05263	0.0082	22,722,096	103,248,109	0.0644	108,528,390	0.0644	0.0000	Total Population: 30 to 34 years
Total Population: 35 to 39 years	22,706,664	0.0807	20,179,642	0.0654	1.04119	0.0153	21,971,022	104,271,146	0.0651	109,606,888	0.0651	0.0000	Total Population: 35 to 39 years
Total Population: 40 to 44 years	22,441,863	0.0797	20,890,964	0.0677	1.01856	0.0121	20,332,501	100,036,852	0.0624	105,163,906	0.0624	0.0000	Total Population: 40 to 44 years
Total Population: 45 to 49 years	20,092,404	0.0714	22,708,591	0.0736	1.00008	0.0022	20,181,355	99,194,021	0.0619	104,279,995	0.0619	0.0000	Total Population: 45 to 49 years
Total Population: 50 to 54 years	17,585,548	0.0625	22,298,125	0.0722	0.99360	0.0097	20,757,159	94,555,892	0.0590	99,396,126	0.0590	0.0000	Total Population: 50 to 54 years
Total Population: 55 to 59 years	13,469,237	0.0479	19,664,805	0.0637	0.97872	0.0158	22,225,315	92,363,286	0.0576	97,083,011	0.0576	0.0000	Total Population: 55 to 59 years
Total Population: 60 to 64 years	10,805,447	0.0384	16,817,924	0.0545	0.95635	0.0161	21,324,793	86,029,156	0.0537	90,428,448	0.0537	0.0000	Total Population: 60 to 64 years
Total Population: 65 to 69 years	9,533,545	0.0339	12,435,263	0.0403	0.92323	0.0064	18,155,225	81,113,547	0.0506	85,272,963	0.0506	0.0000	Total Population: 65 to 69 years
Total Population: 70 to 74 years	8,857,441	0.0315	9,278,166	0.0301	0.85866	0.0014	14,440,818	70,262,346	0.0438	73,869,483	0.0438	0.0000	Total Population: 70 to 74 years
Total Population: 75 to 79 years	7,415,813	0.0264	7,317,795	0.0237	0.76758	0.0026	9,545,107	59,228,785	0.0370	62,261,447	0.0370	0.0000	Total Population: 75 to 79 years
Total Population: 80 to 84 years	4,945,367	0.0176	5,743,327	0.0186	0.64842	0.0010	6,016,133	43,347,511	0.0270	45,559,392	0.0270	0.0000	Total Population: 80 to 84 years
Total Population: 85 years and over	4,239,587	0.0151	5,493,433	0.0178	0.33091	0.0027	6,139,970	47,125,500	0.0294	49,538,482	0.0294	0.0000	Total Population: 85 years and over
Total Population	281,421,906	1.0000	308,745,538	1.0000	1.09709	0.1130	336,410,602	1,602,664,738	1.0000	1,684,718,091	1.0000	0.0000	Total Population

Table 4. The Population of Whitman County, Washington at start (2000-10) and at achieving Stability (2280-90)

	2000 POPULATION	2000 PROPORTION BY AGE	2010 POPULATION	2010 PROPORTION BY AGE	2000-2010 CCR	ABS Difference	PROJECTED 2020	PROJECTED 2280	2280 PROPORTION BY AGE	PROJECTED 2290	2290 PROPORTION BY AGE	ABS Difference	
Total Population: 0 to 4 years	940	0.0463	1,978	0.0442	0.11408	0.0021	3,910	534,917,525,831	0.0383	1,102,624,564,345	0.0383	0.0000	Total Population: 0 to 4 years
Total Population: 5 to 9 years	971	0.0478	1,810	0.0404	0.23352	0.0074	4,416	660,200,456,282	0.0472	1,360,864,336,756	0.0472	0.0000	Total Population: 5 to 9 years
Total Population: 10 to 14 years	1,012	0.0498	1,789	0.0400	1.90319	0.0099	3,765	493,892,894,413	0.0353	1,018,050,482,672	0.0353	0.0000	Total Population: 10 to 14 years
Total Population: 15 to 19 years	2,696	0.1327	6,072	0.1356	6.25335	0.0029	11,319	2,002,801,978,465	0.1433	4,128,462,585,527	0.1433	0.0000	Total Population: 15 to 19 years
Total Population: 20 to 24 years	4,431	0.2181	11,394	0.2545	11.25889	0.0364	20,142	2,697,632,303,153	0.1930	5,560,687,390,253	0.1930	0.0000	Total Population: 20 to 24 years
Total Population: 25 to 29 years	1,368	0.0673	3,621	0.0809	1.34310	0.0135	8,155	1,304,993,320,308	0.0934	2,689,965,120,186	0.0934	0.0000	Total Population: 25 to 29 years
Total Population: 30 to 34 years	1,144	0.0563	2,324	0.0519	0.52449	0.0044	5,976	686,421,383,811	0.0491	1,414,871,918,873	0.0491	0.0000	Total Population: 30 to 34 years
Total Population: 35 to 39 years	1,106	0.0544	1,806	0.0403	1.32018	0.0141	4,780	835,775,205,629	0.0598	1,722,820,129,004	0.0598	0.0000	Total Population: 35 to 39 years
Total Population: 40 to 44 years	1,140	0.0561	1,864	0.0416	1.62937	0.0145	3,787	542,575,866,720	0.0388	1,118,434,842,153	0.0388	0.0000	Total Population: 40 to 44 years
Total Population: 45 to 49 years	1,013	0.0499	2,003	0.0447	1.81103	0.0051	3,271	734,285,642,588	0.0525	1,513,614,590,303	0.0525	0.0000	Total Population: 45 to 49 years
Total Population: 50 to 54 years	912	0.0449	2,212	0.0494	1.94035	0.0045	3,617	510,765,187,540	0.0365	1,052,787,558,934	0.0365	0.0000	Total Population: 50 to 54 years
Total Population: 55 to 59 years	766	0.0377	1,967	0.0439	1.94176	0.0062	3,889	691,710,962,376	0.0495	1,425,804,401,747	0.0495	0.0000	Total Population: 55 to 59 years
Total Population: 60 to 64 years	569	0.0280	1,679	0.0375	1.84101	0.0095	4,072	456,184,878,990	0.0326	940,323,190,658	0.0326	0.0000	Total Population: 60 to 64 years
Total Population: 65 to 69 years	472	0.0232	1,343	0.0300	1.75326	0.0068	3,449	588,282,752,011	0.0421	1,212,751,726,464	0.0421	0.0000	Total Population: 65 to 69 years
Total Population: 70 to 74 years	432	0.0213	885	0.0198	1.55536	0.0015	2,611	344,218,693,674	0.0246	709,531,841,663	0.0246	0.0000	Total Population: 70 to 74 years
Total Population: 75 to 79 years	454	0.0223	716	0.0160	1.51695	0.0064	2,037	432,964,000,554	0.0310	892,395,022,119	0.0310	0.0000	Total Population: 75 to 79 years
Total Population: 80 to 84 years	360	0.0177	584	0.0130	1.35185	0.0047	1,196	225,781,295,157	0.0162	465,332,678,485	0.0161	0.0000	Total Population: 80 to 84 years
Total Population: 85 years and over	529	0.0260	729	0.0163	0.54281	0.0098	1,101	235,449,132,232	0.0168	485,381,785,533	0.0168	0.0000	Total Population: 85 years and over
Total Population	20,315	1.0000	44,776	1.0000		0.1595	91,494	13,978,853,479,733	1.0000	28,814,704,165,673	1.0000	0.0000	Total Population

Table 5. The Population of Whitman County, Washington at start (2000-10) and at achieving Stability (2410-20) when the USA CCRs are applied

	2000 POPULATION	2000 PROPORTION BY AGE	2010 POPULATION	2010 PROPORTION BY AGE	USA 2000-2010 CCR	ABS Difference	PROJECTED 2410	2410 PROPORTION BY AGE	PROJECTED 2420	2420 PROPORTION BY AGE	ABS Difference	
Total Population: 0 to 4 years	940	0.0463	1,978	0.0442	0.32245	0.0021	29,157	0.0627	30,652	0.0627	0.0000	Total Population: 0 to 4 years
Total Population: 5 to 9 years	971	0.0478	1,810	0.0404	0.33226	0.0074	30,159	0.0648	31,700	0.0648	0.0000	Total Population: 5 to 9 years
Total Population: 10 to 14 years	1,012	0.0498	1,789	0.0400	1.07830	0.0099	29,914	0.0643	31,440	0.0643	0.0000	Total Population: 10 to 14 years
Total Population: 15 to 19 years	2,696	0.1327	6,072	0.1356	1.07255	0.0029	30,772	0.0662	32,347	0.0662	0.0000	Total Population: 15 to 19 years
Total Population: 20 to 24 years	4,431	0.2181	11,394	0.2545	1.05154	0.0364	29,917	0.0643	31,455	0.0643	0.0000	Total Population: 20 to 24 years
Total Population: 25 to 29 years	1,368	0.0673	3,621	0.0809	1.04362	0.0135	30,545	0.0657	32,114	0.0657	0.0000	Total Population: 25 to 29 years
Total Population: 30 to 34 years	1,144	0.0563	2,324	0.0519	1.05263	0.0044	29,962	0.0644	31,492	0.0644	0.0000	Total Population: 30 to 34 years
Total Population: 35 to 39 years	1,106	0.0544	1,806	0.0403	1.04119	0.0141	30,263	0.0651	31,803	0.0651	0.0000	Total Population: 35 to 39 years
Total Population: 40 to 44 years	1,140	0.0561	1,864	0.0416	1.01856	0.0145	29,033	0.0624	30,518	0.0624	0.0000	Total Population: 40 to 44 years
Total Population: 45 to 49 years	1,013	0.0499	2,003	0.0447	1.00008	0.0051	28,782	0.0619	30,266	0.0619	0.0000	Total Population: 45 to 49 years
Total Population: 50 to 54 years	912	0.0449	2,212	0.0494	0.99360	0.0045	27,433	0.0590	28,847	0.0590	0.0000	Total Population: 50 to 54 years
Total Population: 55 to 59 years	766	0.0377	1,967	0.0439	0.97872	0.0062	26,803	0.0576	28,169	0.0576	0.0000	Total Population: 55 to 59 years
Total Population: 60 to 64 years	569	0.0280	1,679	0.0375	0.95635	0.0095	24,972	0.0537	26,236	0.0537	0.0000	Total Population: 60 to 64 years
Total Population: 65 to 69 years	472	0.0232	1,343	0.0300	0.92323	0.0068	23,544	0.0506	24,745	0.0506	0.0000	Total Population: 65 to 69 years
Total Population: 70 to 74 years	432	0.0213	885	0.0198	0.85866	0.0015	20,386	0.0438	21,442	0.0439	0.0000	Total Population: 70 to 74 years
Total Population: 75 to 79 years	454	0.0223	716	0.0160	0.76758	0.0064	17,181	0.0369	18,072	0.0370	0.0000	Total Population: 75 to 79 years
Total Population: 80 to 84 years	360	0.0177	584	0.0130	0.64842	0.0047	12,578	0.0270	13,219	0.0270	0.0000	Total Population: 80 to 84 years
Total Population: 85 years and over	529	0.0260	729	0.0163	0.33091	0.0098	13,679	0.0294	14,374	0.0294	0.0000	Total Population: 85 years and over
	20,315	1.0000	44,776	1.0000		0.1595	465,079	1.0000	488,892	1.0000	0.0000	Total Population

Table 6. Difference in Proportional Population by Age for the US At Stability and Whitman County at Stability using US CCRs

	WHITMAN COUNTY 2420 PROPORTION BY AGE	USA 2340 PROPORTION BY AGE	DIFFERENCE
Total Population: 0 to 4 years	0.0627	0.0627	0.0000
Total Population: 5 to 9 years	0.0648	0.0648	0.0000
Total Population: 10 to 14 years	0.0643	0.0643	0.0000
Total Population: 15 to 19 years	0.0662	0.0662	0.0000
Total Population: 20 to 24 years	0.0643	0.0643	0.0000
Total Population: 25 to 29 years	0.0657	0.0657	0.0000
Total Population: 30 to 34 years	0.0644	0.0644	0.0000
Total Population: 35 to 39 years	0.0651	0.0651	0.0000
Total Population: 40 to 44 years	0.0624	0.0624	0.0000
Total Population: 45 to 49 years	0.0619	0.0619	0.0000
Total Population: 50 to 54 years	0.0590	0.0590	0.0000
Total Population: 55 to 59 years	0.0576	0.0576	0.0000
Total Population: 60 to 64 years	0.0537	0.0537	0.0000
Total Population: 65 to 69 years	0.0506	0.0506	0.0000
Total Population: 70 to 74 years	0.0439	0.0438	0.0000
Total Population: 75 to 79 years	0.0370	0.0370	0.0000
Total Population: 80 to 84 years	0.0270	0.0270	0.0000
Total Population: 85 years and over	0.0294	0.0294	0.0000
SUM	**1.0000**	**1.0000**	**0.0000**

TABLE 7. INPUT DATA FOR THE REGRESSION MODEL		
POPULATION	**INITIAL STABILITY INDEX**	**YEARS TO STABILITY, *S* =0.01**
PIMA CO, AZ	0.06099	70
JEFFERSON CO, AR	0.06563	70
TULARE CO, CA	0.03966	80
BROWARD CO, FL	0.08147	110
LAKE CO, IL	0.08442	110
BLACK HAWK CO, IA	0.06886	100
CALVERT CO, MD	0.11430	130
HAMPDEN CO, MA	0.08246	110
MADISON CO, MS	0.07240	60
DOUGLAS CO, NE	0.05269	70
BRONX CO, NY	0.06185	120
ROCKLAND CO, NY	0.05063	70
FRANKLIN CO, OH	0.05076	70
MULTNOMAH CO, OR	0.06130	100
SCHUYLKILL CO, PA	0.06444	70
SEVIER CO, TN	0.05636	70
YAKIMA CO, WA	0.04223	60

Table 8. Estimated and Actual Years to S =0.01

Population	Initial S	Actual Years to S = 0.01	Estimated Years to S =0.01 using the regression models	Difference (estimate - actual)	Percent Difference
Alaska	0.10573	120	123	3	2.50%
United States	0.0565	60	52	-8	-13.19%
Whitman County, WA	0.07903	90	71	-19	-20.56%

TABLE 9. THE MOST RECENT ENTRIES FOR 67 COUNTRIES IN KEYFITZ & FLIEGER(1968)

COUNTRY	YEAR	OBSERVED RATE OF NATURAL INCREASE	INTRINSIC RATE
MAURITIUS	1965	26.8300	31.1491
REUNION	1961	32.3800	33.0756
SOUTH AFRICA COLORED POP	1960	31.1500	32.1806
SOUTH AFRICA WHITE POP	1960	16.1600	18.3173
TOGO	1961	25.5600	18.3170
BARBADOS	1965	18.2600	18.0947
CANADA (EXL NEWFOUNDLAND)	1965	13.5200	14.6668
ONTARIO (CANADA	1960-62	19.0200	18.7584
COSTA RICA	1963	41.3100	41.5200
DOMINICAN REPUBLIC	1960	27.3400	24.4959
EL SALVADOR	1950	33.9600	28.0790
GRENADA	1960	33.4200	39.1496
HOUNDURAS	1965	34.4400	34.0591
JAMAICA	1956	27.8200	27.8203
MARTINIQUE	1963	25.0300	28.2939
MEXICO	1962	33.7900	33.6501
PANAMA (EXL CANAL ZONE & TRIBES)	1962	34.2200	34.6460
PUERTO RICO	1965	23.5800	22.3987
ST KITTS-NEVIS & ANGUILLA	1960	29.1400	37.0359
SANTA LUCIA	1960	34.1200	35.1625
TRINIDAD & TOBAGO	1959-61	30.3000	33.0584
USA (WITH ADJUSTED BIRTHS)	1965	10.1800	12.6505
ARGENTINA	1961	14.1700	8.4115
BRITISH GUIANA (BELIZE)	1956	31.9500	36.0682
CHILE	1964	21.6000	21.0531
COLUMBIA	1964	27.9100	28.3471
FRENCH GUIANA	1961	17.3500	26.3188
PERU	1961	25.0000	23.4555
VENEZUELA	1963	36.23	36.6998
CEYLON	1962	26.99	26.3984
CHINA (TAIWAN)	1965	27.22	27.981
CYPRESS	1960	19.72	16.833
ISRAEL	1963	15.91	16.0788
JAPAN	1963	10.31	-2.769
SINGAPORE	1962	28.45	31.7108
THAILAND	1960	26.42	22.1485
ALBANIA	1955	29.36	29.3867
AUSTRIA	1965	4.91	8.5456
BELGIUM	1963	4.57	8.0839
BULGARIA	1965	7.17	-1.5469
CZECHOSLOVAKIA	1964	7.57	6.4485
DENMARK	1964	7.74	7.6104
FINLAND	1965	7.25	4.5477
FRANCE	1965	6.58	10.3859
GERMANY (EAST)	1964	3.87	6.0057
GERMANY (WEST, INC W. BERLIN)	1965	6.21	5.5359
GREECE	1965	9.85	1.2845
HUNGARY	1965	2.45	-7.0812
ICELAND	1962	19.04	23.2193
IRELAND	1960-62	9.6	19.1095
ITALY	1964	10.09	6.3874
LUXEMBOURG	1963	3.59	1.9932
MALTA	1965	8.23	4.7856
NETHERLANDS	1965	11.97	12.5215
NORWAY	1964	8.25	11.9853
POLAND	1962	11.88	7.3215
PORTUGAL	1965	12.47	9.7744
ROMANIA	1965	6.04	-4.2811
SPAIN	1963	12.42	9.1676
SWEDEN	1965	5.77	5.1733
SWITZERLAND	1964	10.27	7.7072
UNITED KINGDOM , ENGLAND & WALES	1963	5.98	10.8473
UNITED KINGDOM, SCOTLAND	1963	7.14	12.6657
YUGOSLAVIA	1961	13.71	6.8722
AUSTRALIA	1965	10.84	12.2941
FIJI ISLANDS	1964	31.79	30.838
NEW ZEALAND	1965	14.12	13.5453

TABLE 10. INITIAL RATE OF INCREASE AND INTRINSIC R

POPULATION	INITIAL R	INTRINSIC R
PIMA CO, AZ	0.01500	0.01002
JEFFERSON CO, AR	-0.00847	-0.01490
TULARE CO, CA	0.01836	0.01744
BROWARD CO, FL	0.00742	0.00628
LAKE CO, IL	0.00878	0.00128
BLACK HAWK CO, IA	0.00238	0.00028
CALVERT CO, MD	0.01740	0.00994
HAMPDEN CO, MA	0.00158	-0.00201
MADISON CO, MS	0.02429	0.01708
DOUGLAS CO, NE	0.01093	0.00881
BRONX CO, NY	0.00386	0.00053
ROCKLAND CO, NY	0.00834	0.00832
FRANKLIN CO, OH	0.00847	0.00540
MULTNOMAH CO, OR	0.01073	0.00620
SCHUYLKILL CO, PA	-0.00137	-0.00521
SEVIER CO, TN	0.02335	0.01810
YAKIMA CO, WA	0.00887	0.00631

TABLE 11 ESTIMATED INTRINSIC R AND ACTUAL INTRINSIC R FOR THREE POPULATIONS

Population	Initial r	est r'	actual r'	Difference	Percent Difference
ALASKA	0.0125	0.0083	0.0079	-0.0004	-4.94%
USA	0.0093	0.0053	0.0048	-0.0005	-10.80%
WHITMAN COUNTY	0.0790	0.0720	0.0758	0.0038	5.02%

Figure 1. Stability Index (***S***) over time (in Years) as the Alaskan Population moves to Stability

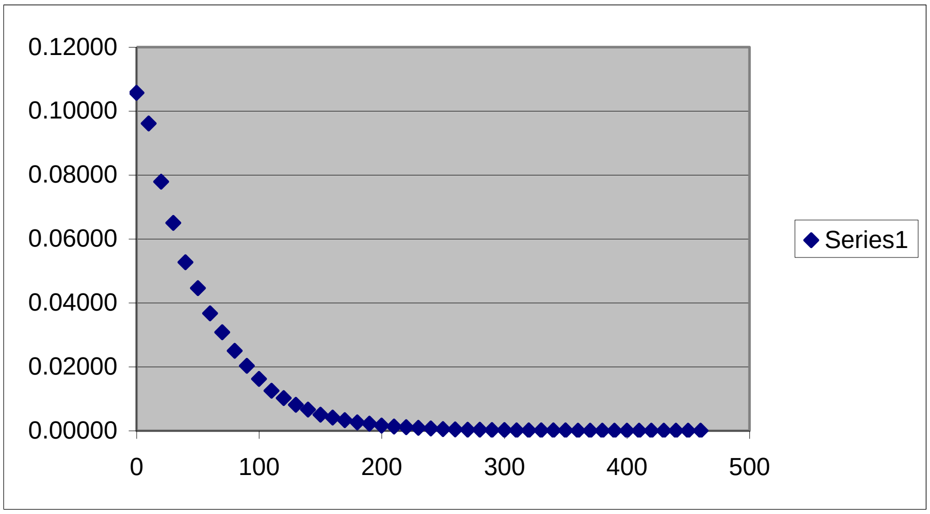

Figure 2. Age Distribution of Alaska in 2010

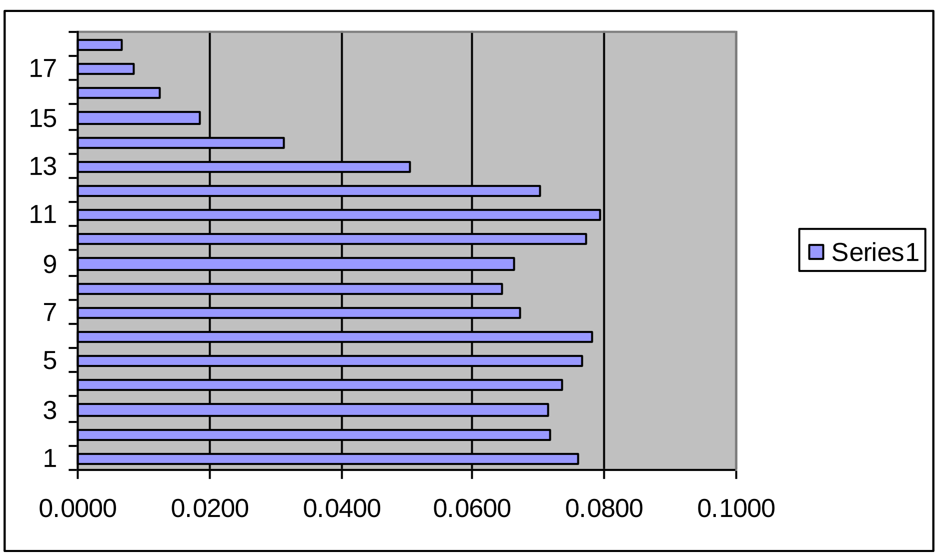

Figure 3. Age Distribution of Alaska in 2470

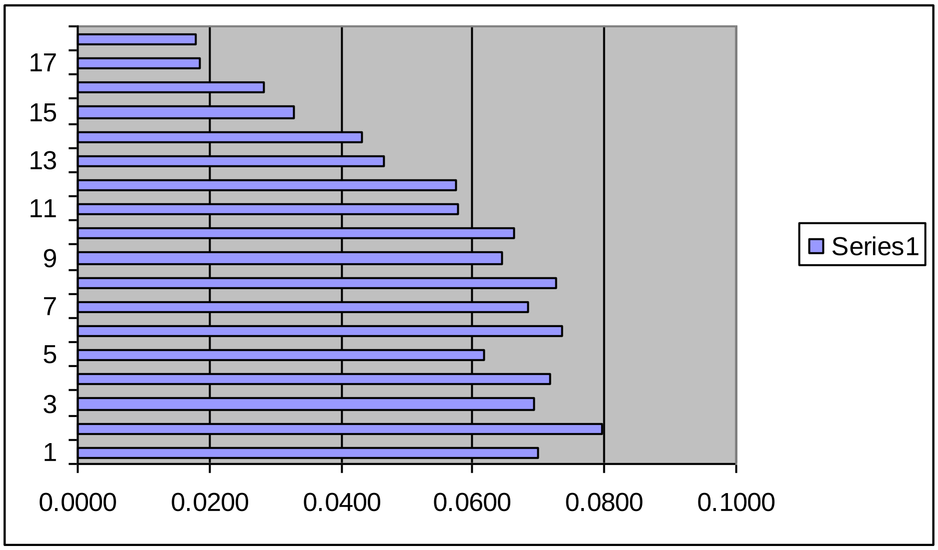

Figure 4. Stability Index (***S***) over time (in Years) as the U.S. Population moves to Stability

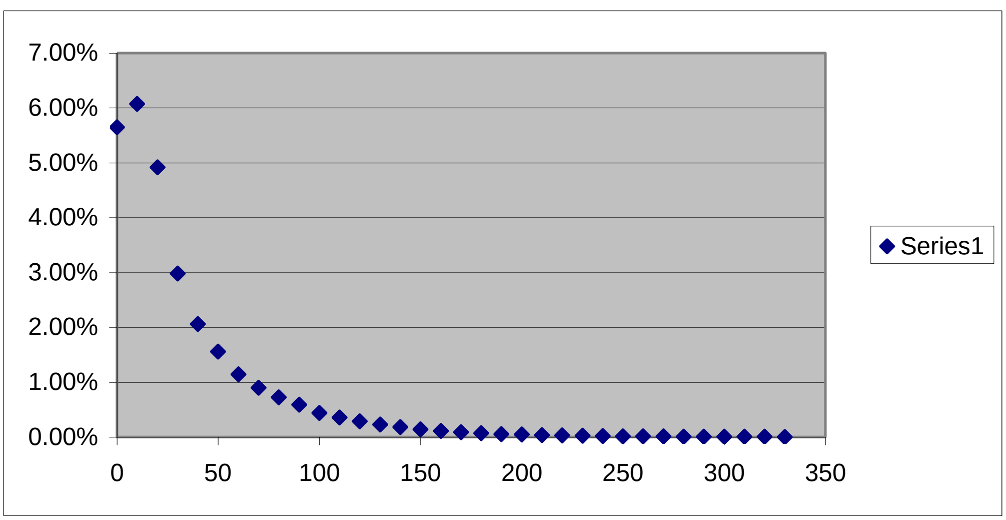

Figure 5. The Natural Logarithm of ***S*** over time (in Years) as the U.S. Population moves to Stability

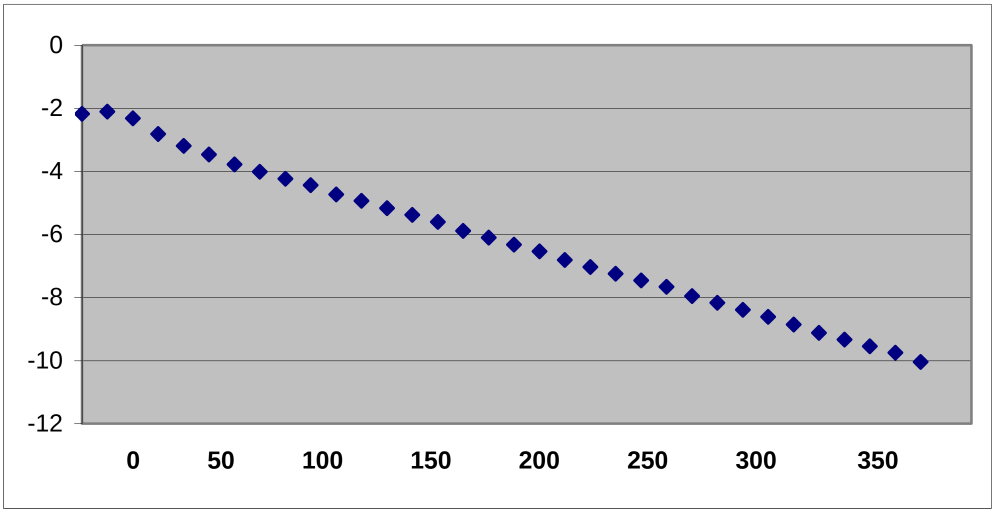

Figure 6. Age Distribution of the U.S. in 2000

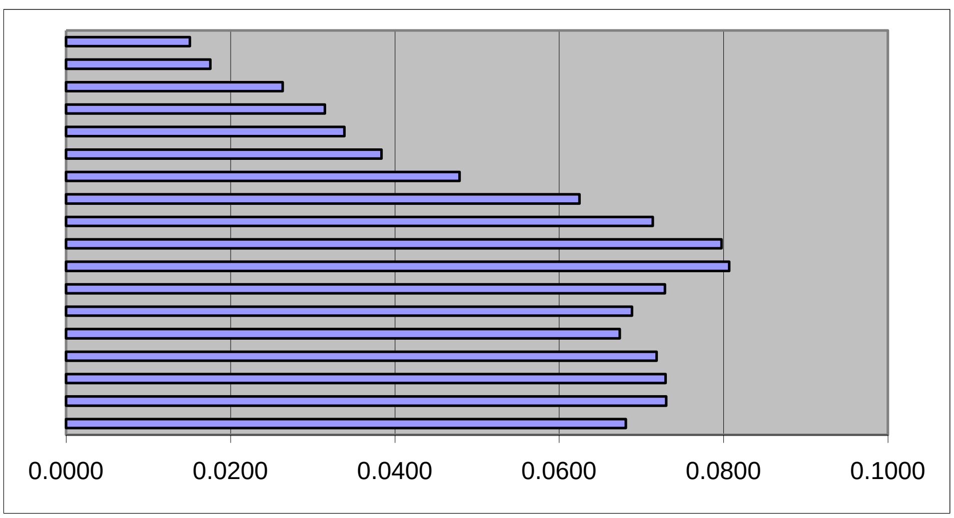

Figure 7. Age Distribution of the U.S. in 2340

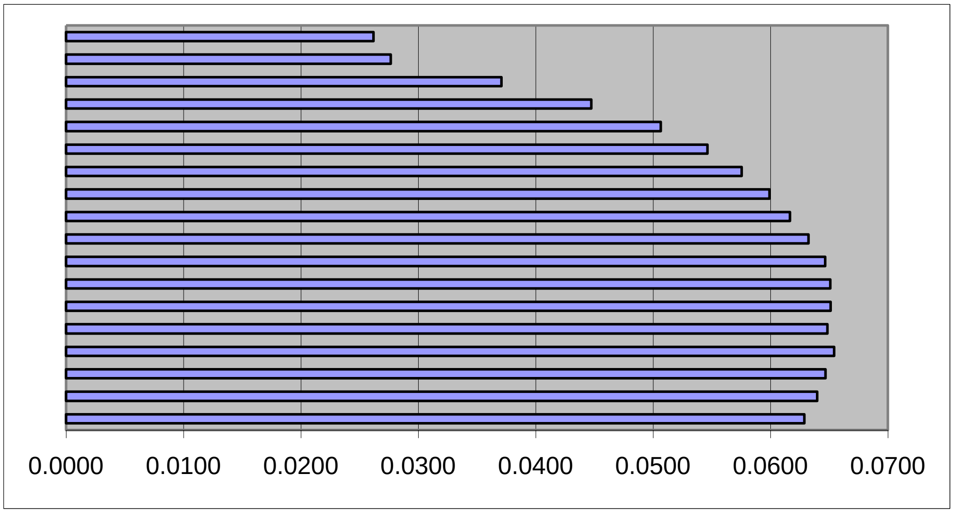

Figure 8. Stability Index (***S***) over time (in Years) as the Whitman County Population moves to Stability

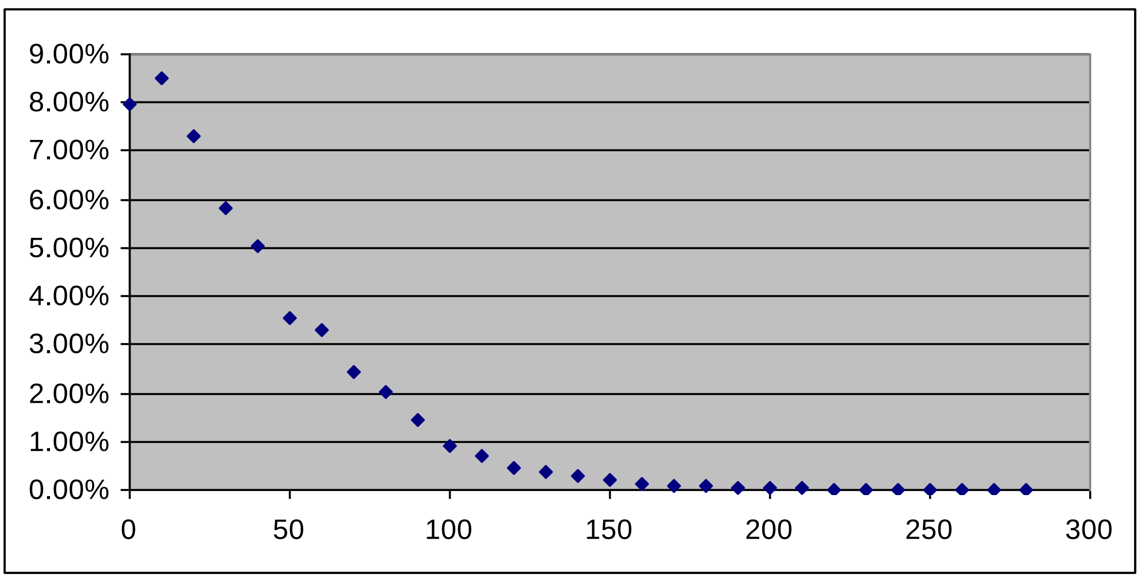

Figure 9. The Natural Logarithm of ***S*** over time (in Years) as the Whitman County Population moves to Stability

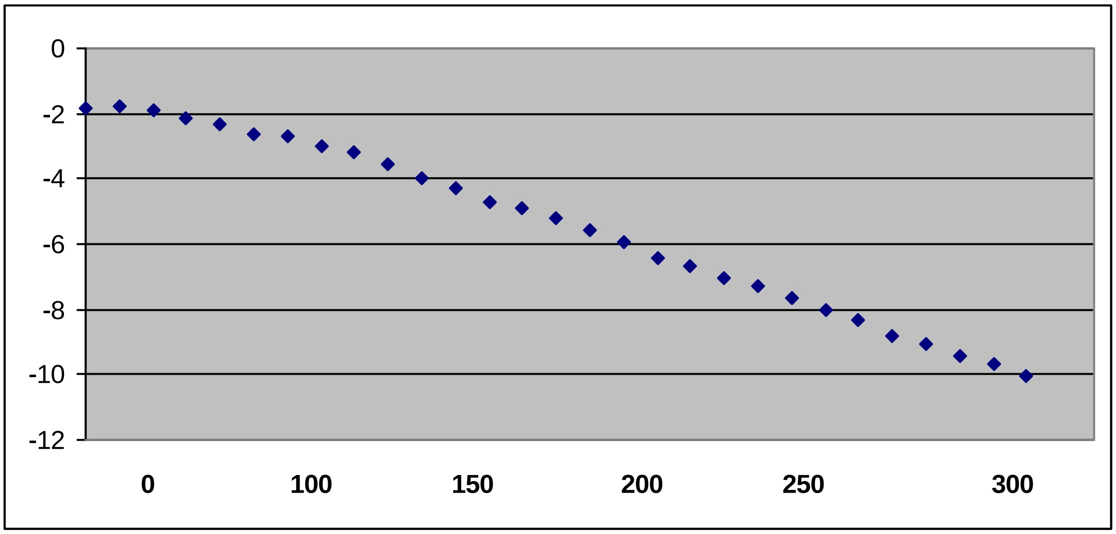

Figure 10. Age Distribution of Whitman County in 2000

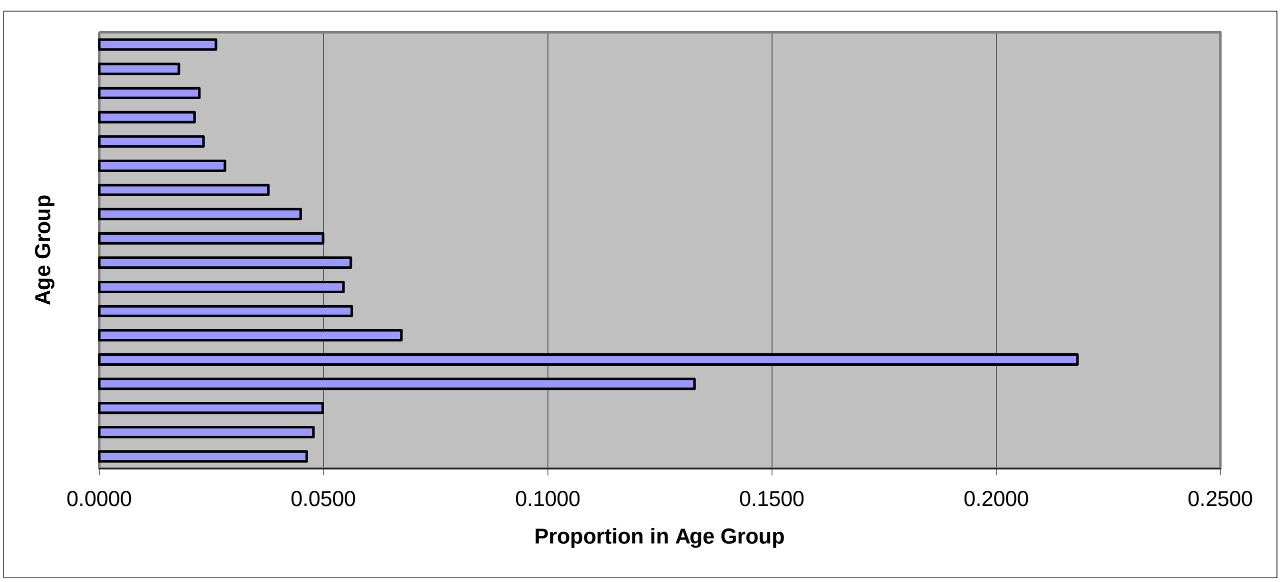

Figure 11. Age Distribution of Whitman County in 2290

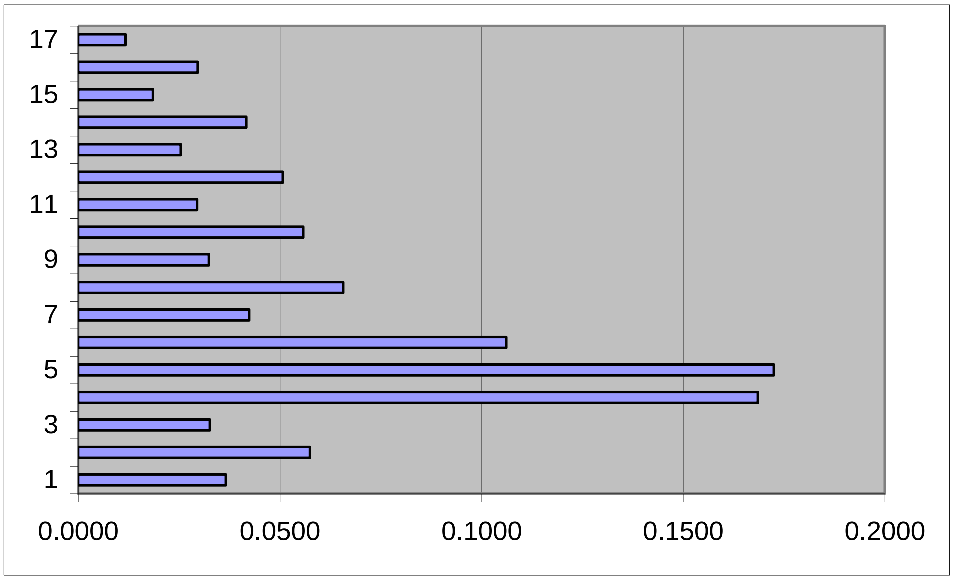

Figure 12. Age Distribution of Whitman County in 2420 when USA CCRs are applied

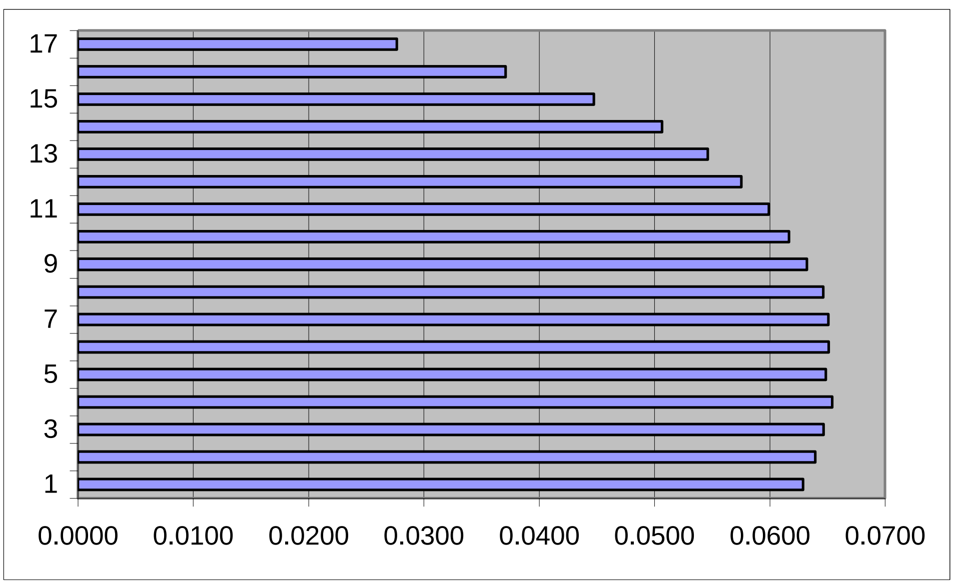

Figure 13. Relationship between Initial *S* and Years to Approximate Stability (*S*=0.01)

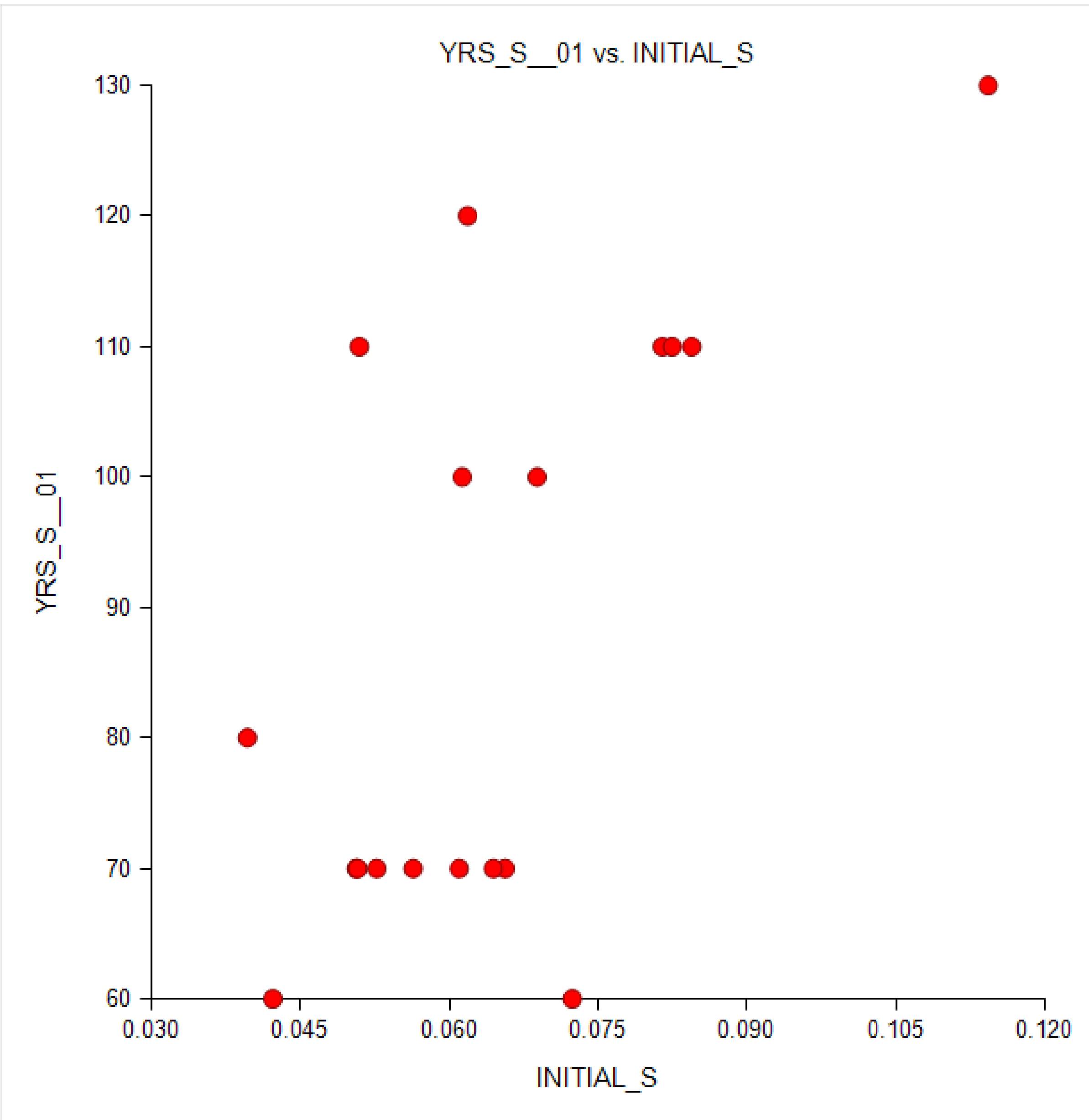

Figure 14. Relationship between initial r and intrinsic r, 67 populations

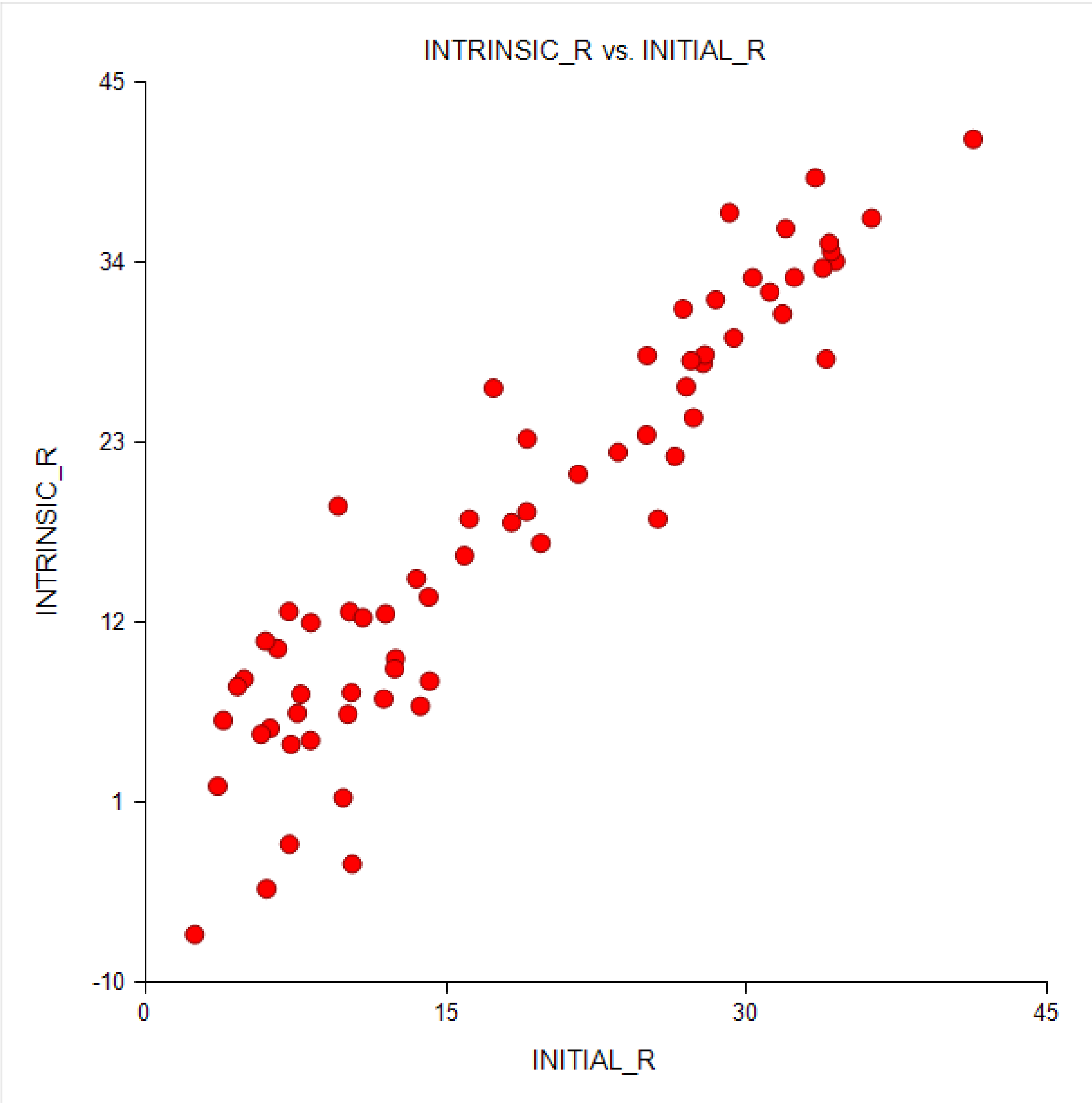

Figure 15. Relationship between initial r and intrinsic r, 18 US Counties

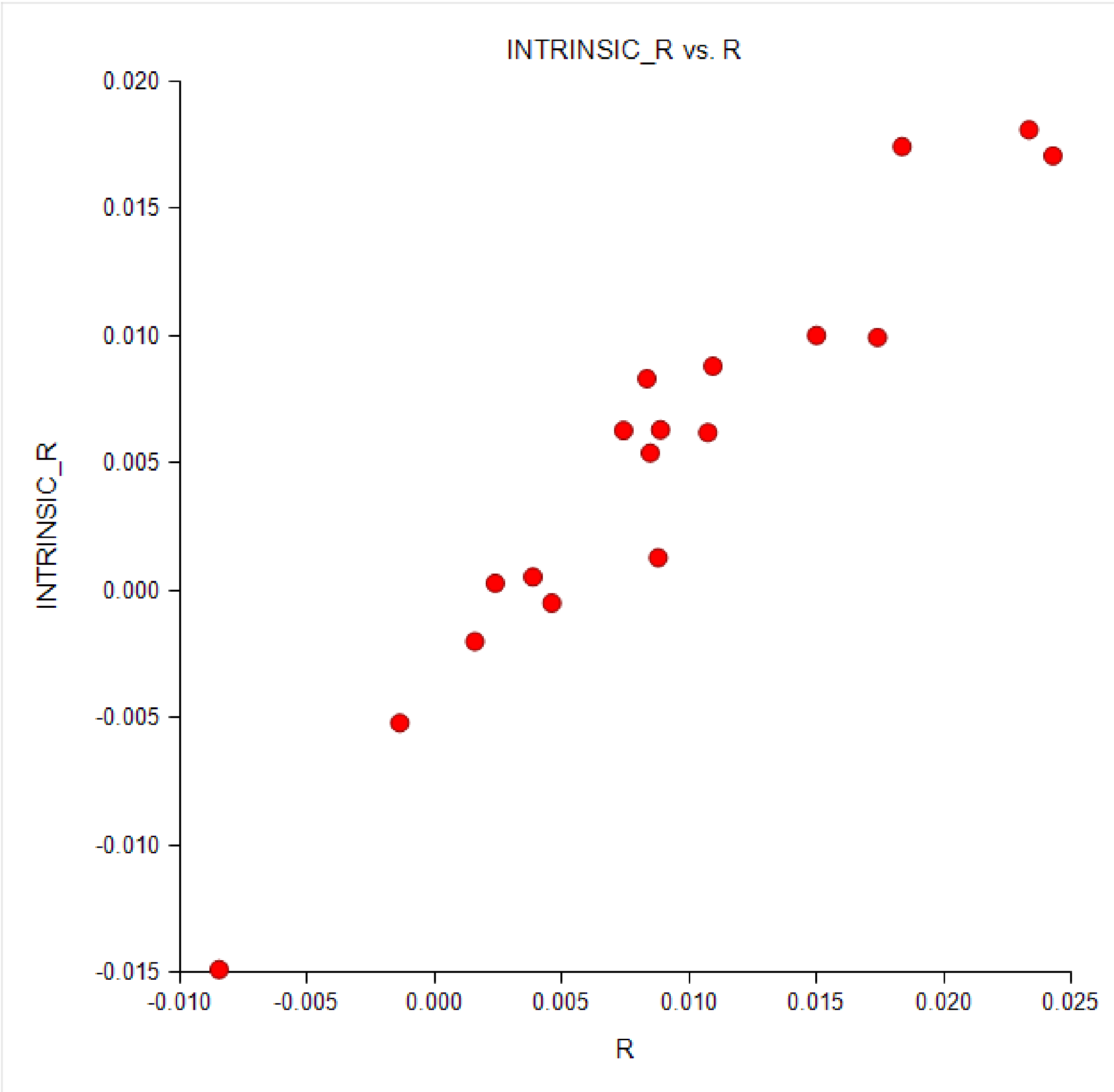

Exhibit 1. The 18 counties used in the Regression Analysis

Pima County, AZ	Madison County, MS
Jefferson County, AR	Douglas County, NE
San Francisco County, CA	Bronx County, NY
Tulare County, CA	Rockland County, NY
Broward County, FL	Franklin County, OH
Lake County, IL	Multnomah County, OR
Black Hawk County, IA	Schuylkill County, PA
Calvert County, MD	Sevier County, TN
Hampden County, MA	Yakima County, WA